OTHER COLLEGE

FYJC - MATHEMATICS & STATISTICS

PAPER - I

LIMITS

Compiled & Conducted @ JKSC

LIMITS OF ALGEBRIAC FUNCTIONS

Q SET - 1

01. $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x(x-5) + 6}$ *ans : –3*

02. $\lim_{x \to 3} \dfrac{x^2 - 5x + 6}{2x^2 - 3x - 9}$ *ans : 1/9*

03. $\lim_{x \to 1} \dfrac{x^2 - 8x + 7}{7x^2 - 6x - 1}$ ans : –3/4

04. $\lim_{x \to 1} \dfrac{3x(x^2 - 7x + 6)}{(x + 2)(x^2 - 4x + 3}$ ans : 5/2

05. $\lim_{x \to -4} \dfrac{x^2 + 3x - 4}{x^2 + 9x + 20}$ ans : –5

06. $\lim_{x \to 2} \dfrac{x^4 - 16}{x^2 - 5x + 6}$ ans : –32

07. $\lim_{x \to 3} \left(\dfrac{1}{x - 3} - \dfrac{3}{x^2 - 3x}\right)$ ans : 1/3

08. $\lim_{x \to 5} \left(\dfrac{1}{x - 5} - \dfrac{5}{x^2 - 5x}\right)$ *ans : 1/5*

09. $\lim_{x \to 2} \left(\dfrac{1}{x - 2} - \dfrac{4}{x^3 - 2x^2}\right)$ *ans : 1*

10.
$\lim_{x \to 4} \left(\dfrac{1}{x^2 - 3x - 4} + \dfrac{1}{x^2 - 13x + 36}\right)$

11. $\lim_{x \to 3} \left(\dfrac{1}{x^2 - 11x + 24} + \dfrac{1}{x^2 - x - 6}\right)$

ans : –2/25

12. $\lim_{x \to 2} \left(\dfrac{1}{x - 2} - \dfrac{3}{x^2 - x - 2}\right)$

ans : 1/3

13. $\lim_{x \to 3} \left(\dfrac{1}{x - 3} - \dfrac{9x}{x^3 - 27}\right)$ *ans : 0*

14. $\lim_{x \to 2} \left(\dfrac{1}{x - 2} + \dfrac{6x}{8 - x^3}\right)$ *ans : 0*

Q SET - 2

01. $\lim_{x \to 1} \dfrac{\sqrt{x + 4} - \sqrt{5}}{x - 1}$ *ans : 1/2 √5*

02. $\lim_{x \to 1} \dfrac{\sqrt{x^2 + 1} - \sqrt{2}}{x - 1}$ *ans : 1/√2*

03. $\lim_{x \to 3} \dfrac{\sqrt{x^2 + 1} - \sqrt{10}}{x - 3}$ *ans : 3/√10*

04. $\lim_{x \to 3} \dfrac{x + 1 - \sqrt{x + 13}}{x - 3}$ *ans : 7/8*

05. $\lim_{x \to 4} \dfrac{x^2 + x - 20}{\sqrt{3x + 4} - 4}$ *ans : 24*

06. $\lim_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x\sqrt{2(2 + x)}}$ *ans : 1/4 √2*

07. $\lim_{x \to 0} \dfrac{\sqrt{1 + x + x^2} - 1}{x}$ ans : $3/\sqrt{10}$

08. $\lim_{x \to 3} \dfrac{x^3 - 27}{\sqrt{x^2 + 7} - 4}$ *ans : 36*

09. $\lim_{x \to 4} \dfrac{2 - \sqrt{8 - x}}{1 - \sqrt{5 - x}}$ *ans : 1/2*

10. $\lim_{x \to 7} \dfrac{4 - \sqrt{9 + x}}{1 - \sqrt{8 - x}}$ *ans : −1/4*

11. $\lim_{x \to 2} \dfrac{x^2 + \sqrt{x + 2} - 6}{x^2 - 4}$ *ans : 17/16*

12. $\lim_{x \to 3} \dfrac{x^2 + \sqrt{x + 6} - 12}{x^2 - 9}$ *ans : 37/36*

13. $\lim_{x \to 4} \dfrac{x^2 + \sqrt{x + 5} - 19}{x^2 - 16}$ *ans : 49/48*

Q SET - 3

01. $\lim_{x \to 2} \dfrac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$ ans : 15/11

02. $\lim_{x \to 3} \dfrac{x^3 - x - 24}{x^3 + x^2 - 36}$ ans : 26/33

03. $\lim_{x \to 3} \dfrac{x^3 - 4x - 15}{x^3 + x^2 - 6x - 18}$ ans : 23/27

04. $\lim_{x \to 1} \dfrac{3x^3 + 4x^2 - 6x - 1}{2x^3 - x - 1}$ ans : 11/5

05. $\lim_{x \to 2} \dfrac{x^3 - 3x^2 + 4}{x^3 - 2x^2 - 4x + 8}$ ans : 3/4

06. $\lim_{x \to 1} \dfrac{x^3 - x^2 - x + 1}{x^3 + x^2 - 5x + 3}$ ans : 1/2

07. $\lim_{x \to 3} \dfrac{x^3 - 3x^2 - 9x + 27}{x^3 - 6x^2 - 9x}$ ans : 2

Q SET - 4

01. $\lim_{x \to a} \dfrac{x^{25} - a^{25}}{x^{15} - a^{15}}$ *ans : $5a^{10}/3$*

02. $\lim_{x \to a} \dfrac{x^7 - a^7}{x^{11} - a^{11}}$ *ans : $7/11a^4$*

03. $\lim_{x \to 4} \dfrac{x^3 - 64}{x^2 - 16}$ *ans : 6*

04. $\lim_{x \to 2} \dfrac{x^6 - 64}{x^{10} - 1024}$ *ans : 3/80*

05. $\lim_{x \to a} \dfrac{x^{-3} - a^{-3}}{x^{-7} - a^{-7}}$ *ans : $3a^4/7$*

06. $\lim_{x \to 3} \dfrac{x^{-4} - 3^{-4}}{x^{-7} - 3^{-7}}$ *ans : 108 / 7*

07. $\lim_{x \to 3} \dfrac{x^{1/4} - 3^{1/4}}{x^{1/3} - 3^{1/3}}$ *ans:* $3^{11/12}/7$

08. $\lim_{x \to k} \dfrac{x^8 - k^8}{x - k} = 8$, find k

09. $\lim_{x \to k} \dfrac{x^5 - k^5}{x - k} = 80$, find k

10. $\lim_{x \to 1} \dfrac{x + x^2 + x^3 - 3}{x - 1}$ *ans : 6*

11. $\lim_{x \to 3} \dfrac{x + x^2 + x^3 - 39}{x - 3}$ *ans : 34*

Q SET - 5

01. Discuss whether the limit exist as $x \to 3$

$$f(x) = \begin{cases} x^2 + x + 1 , & 2 \le x \le 3 \\ 2x + 1 , & 3 < x \le 4 \end{cases}$$

02. Discuss whether the limit exist as $x \to 3$

$$f(x) = \begin{cases} x^2 - 3x + 7 , & x \le 3 \\ x + 1 , & 3 < x \end{cases}$$

03. Discuss whether the limit exist as $x \to 2$

$$f(x) = \begin{cases} 4x + 3 , & x \le 2 \\ 2x^2 + 3 , & x > 2 \end{cases}$$

04. Discuss whether the limit exist as $x \to 0$

$$f(x) = \begin{cases} x^2 + 1 , & 0 \le x \le 2 \\ 2\sqrt{x^2 + 1} - 1 , & -2 \le x < 0 \end{cases}$$

05. Discuss whether the limit exist as $x \to 1$

$$f(x) = \begin{cases} 5x - 1 , & x \le 1 \\ \dfrac{2x^2 - 1}{x + 5} , & x > 1 \end{cases}$$

SOLUTION TO Q SET - 1

01. $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x(x-5) + 6}$

$-2 \quad +1$

$= \lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 5x + 6}$

$-2 \quad -3$

$= \lim\limits_{x \to 2} \dfrac{x^2 - 2x + 1x - 2}{x^2 - 2x - 3x + 6}$

$= \lim\limits_{x \to 2} \dfrac{x(x-2) + 1(x-2)}{x(x-2) - 3(x-2)}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}(x-3)}$ **CUT**

$(x \to 2 \,;\, x \neq 2 \;\therefore x - 2 \neq 0)$

$= \lim\limits_{x \to 2} \dfrac{x+1}{x-3}$ **COPY**

$= \dfrac{2+1}{2-3}$ **PASTE**

$= -3$

02. $\lim\limits_{x \to 3} \dfrac{x^2 - 5x + 6}{2x^2 - 3x - 9}$

$-3 \quad -2$

$= \lim\limits_{x \to 3} \dfrac{x^2 - 5x + 6}{2x^2 - 3x - 9}$

$-6 \quad +3$

$= \lim\limits_{x \to 3} \dfrac{x^2 - 3x - 2x + 6}{2x^2 - 6x + 3x - 9}$

$= \lim\limits_{x \to 3} \dfrac{x(x-3) - 2(x-3)}{2x(x-3) + 3(x-3)}$

$= \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(2x+3)}$ **CUT**

$(x \to 3 \,;\, x \neq 3 \;\therefore x - 3 \neq 0)$

$= \lim\limits_{x \to 3} \dfrac{x-2}{2x+3}$ **COPY**

$= \dfrac{3-2}{2(3)+3}$ **PASTE**

$= \dfrac{1}{9}$

03. $\lim\limits_{x \to 1} \dfrac{x^2 - 8x + 7}{7x^2 - 6x - 1}$

$-1 \quad -7$

$= \lim\limits_{x \to 1} \dfrac{x^2 - 8x + 7}{7x^2 - 6x - 1}$

$-7 \quad +1$

$= \lim\limits_{x \to 1} \dfrac{x^2 - 1x - 7x + 7}{7x^2 - 7x + 1x - 1}$

$= \lim\limits_{x \to 1} \dfrac{x(x-1) - 7(x-1)}{7x(x-1) + 1(x-1)}$

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x-1)}(x-7)}{\cancel{(x-1)}(7x+1)}$ **CUT**

$(x \to 1 \,;\, x \neq 1 \;\therefore x - 1 \neq 0)$

$= \lim\limits_{x \to 1} \dfrac{x-7}{7x+1}$ **COPY**

$= \dfrac{1-7}{7(1)+1}$ **PASTE**

$= \dfrac{-6}{8}$

$= -3/4$

04. $$\lim_{x \to 1} \frac{3x(x^2 - 7x + 6\ )}{(x + 2)(x^2 - 4x + 3}$$

$-1 \quad -6$

$$= \lim_{x \to 1} \frac{3x.(x^2 - 7x + 6)}{(x + 2).(\ x^2 - 4x + 3)}$$

$-3 \quad -1$

$$= \lim_{x \to 1} \frac{3x.(x^2 - 1x - 6x + 6)}{(x + 2).(\ x^2 - 3x - 1x + 3)}$$

$$= \lim_{x \to 1} \frac{3x.(x^2 - 1x - 6x + 6)}{(x + 2).(\ x^2 - 3x - 1x + 3)}$$

$$= \lim_{x \to 1} \frac{3x.[\ x(x - 1) - 6(x - 1)\ ]}{(x + 2).[x(x\ - 3) - 1(x - 3)\ ]}$$

$$= \lim_{x \to 1} \frac{3x.\ \ (x - 1)(x - 6)}{(x + 2).\ (x\ - 1)(x - 3)}$$ **CUT**

$$= \lim_{x \to 1} \frac{3x.\ (x - 6)}{(x + 2).(x - 3)}$$ **COPY**

$$= \frac{3(1)(1 - 6)}{(1 + 2)(1 - 3)}$$ **PASTE**

$$= \frac{3\ (-5)}{3(-2)}$$

$$= 5/2$$

05. $$\lim_{x \to -4} \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$$

$+4 \quad -1$

$$= \lim_{x \to -4} \frac{x^2 + 3x - 4}{x^2 + 9x + 20}$$

$+4 \quad +5$

$$= \lim_{x \to -4} \frac{x^2 + 4x - 1x - 4}{x^2 + 4x + 5x + 20}$$

$$= \lim_{x \to -4} \frac{x(x + 4) - 1(x + 4)}{x(x + 4) + 5(x + 4)}$$

$$= \lim_{x \to -4} \frac{\cancel{(x + 4)}(x - 1)}{\cancel{(x + 4)}(x + 5)}$$ **CUT**

$(x \to -4\ ;\ x \neq -4 \therefore x + 4 \neq 0\ )$

$$= \lim_{x \to -4} \frac{x - 1}{x + 5}$$ **COPY**

$$= \frac{-4 - 1}{-4 + 5}$$ **PASTE**

$$= -5$$

06. $$\lim_{x \to 2} \frac{x^4 - 16}{x^2 - 5x + 6}$$

$$= \lim_{x \to 2} \frac{(x^2 - 4)(x^2 + 4)}{x^2 - 5x + 6}$$

$-2 \quad -3$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 2)\ .\ (x^2 + 4)}{x^2 - 2x - 3x + 6}$$

$$= \lim_{x \to 2} \frac{(x - 2)(x + 2)\ .\ (x^2 + 4)}{x(x - 2)\ - 3(x - 2)}$$

$$= \lim_{x \to 2} \frac{\cancel{(x - 2)}(x + 2)\ .\ (x^2 + 4)}{\cancel{(x - 2)}(x - 3)}$$ **CUT**

$(x \to 2\ ;\ x \neq 2\ \therefore x - 2 \neq 0\ )$

$$= \lim_{x \to 2} \frac{(x + 2)\ .\ (x^2 + 4)}{x - 3}$$ **COPY**

$$= \frac{2 + 2)(4 + 4)}{2 - 3}$$ **PASTE**

$$= \frac{4(8)}{-1} \qquad = -32$$

07. $\lim\limits_{x \to 3} \left(\dfrac{1}{x-3} - \dfrac{3}{x^2-3x} \right)$

$= \lim\limits_{x \to 3} \left(\dfrac{1}{x-3} - \dfrac{3}{x(x-3)} \right)$

$= \lim\limits_{x \to 3} \dfrac{\cancel{x-3}}{x\cancel{(x-3)}}$ **CUT**

$(x \to 3\ ;\ x \neq 3\ \therefore x - 3 \neq 0\ )$

$= \lim\limits_{x \to 3} \dfrac{1}{x}$ **COPY**

$= \dfrac{1}{3}$ **PASTE**

08. $\lim\limits_{x \to 5} \left(\dfrac{1}{x-5} - \dfrac{5}{x^2-5x} \right)$

$= \lim\limits_{x \to 5} \left(\dfrac{1}{x-5} - \dfrac{5}{x(x-5)} \right)$

$= \lim\limits_{x \to 5} \dfrac{\cancel{x-5}}{x\cancel{(x-5)}}$ **CUT**

$= \lim\limits_{x \to 5} \dfrac{1}{x}$ **COPY**

$= \dfrac{1}{5}$ **PASTE**

09. $\lim\limits_{x \to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^3-2x^2} \right)$

$= \lim\limits_{x \to 2} \left(\dfrac{1}{x-2} - \dfrac{4}{x^2(x-2)} \right)$

$= \lim\limits_{x \to 2} \dfrac{x^2-4}{x^2(x-2)}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x+2)}{x^2\cancel{(x-2)}}$ **CUT**

$= \lim\limits_{x \to 2} \dfrac{x+2}{x^2}$ **COPY**

$= \dfrac{2+2}{2^2}$ **PASTE**

$= 1$

10.

$\lim\limits_{x \to 4} \left(\dfrac{1}{x^2-3x-4} + \dfrac{1}{x^2-13x+36} \right)$

$= \lim\limits_{x \to 4} \left(\dfrac{1}{\underbrace{x^2-3x-4}_{-4 \qquad +1}} + \dfrac{1}{\underbrace{x^2-13x+36}_{-9 \qquad -4}} \right)$

$= \lim\limits_{x \to 4} \left(\dfrac{1}{x^2-4x+x-4} + \dfrac{1}{x^2-9x-4x+36} \right)$

$= \lim\limits_{x \to 4} \left(\dfrac{1}{x(x-4)+1(x-4)} + \dfrac{1}{x(x-9)-4(x-9)} \right)$

$= \lim\limits_{x \to 4} \left(\dfrac{1}{(x-4)(x+1)} + \dfrac{1}{(x-9)(x-4)} \right)$

$= \lim\limits_{x \to 4} \dfrac{x-9+x+1}{(x-4)(x+1)(x-9)}$

$= \lim\limits_{x \to 4} \dfrac{2x-8}{(x-4)(x+1)(x-9)}$

$= \lim\limits_{x \to 4} \dfrac{2\cancel{(x-4)}}{\cancel{(x-4)}(x+1)(x-9)}$ **CUT**

$(x \to 4\ ;\ x \neq 4\ \therefore x - 4 \neq 0\ )$

$= \lim\limits_{x \to 4} \dfrac{2}{(x+1)(x-9)}$ **COPY**

$= \dfrac{2}{(4+1)(4-9)}$ **PASTE**

$= \dfrac{2}{(5)(-5)} \quad = \quad -2\ /\ 25$

11.

$$\lim_{x \to 3} \left(\frac{1}{x^2 - 11x + 24} + \frac{1}{x^2 - x - 6} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{\underbrace{x^2 - 11x + 24}_{-3 \quad -8}} + \frac{1}{\underbrace{x^2 - x - 6}_{-3 \quad +2}} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{x^2 - 3x - 8x + 24} + \frac{1}{x^2 - 3x + 2x - 6} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{x(x - 3) - 8(x - 3)} + \frac{1}{x(x-3) + 2(x - 3)} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{(x - 3)(x - 8)} + \frac{1}{(x - 3)(x + 2)} \right)$$

$$= \lim_{x \to 3} \frac{x + 2 + x - 8}{(x - 3)(x - 8)(x + 2)}$$

$$= \lim_{x \to 3} \frac{2x - 6}{(x - 3)(x - 8)(x + 2)}$$

$$= \lim_{x \to 3} \frac{2\cancel{(x - 3)}}{\cancel{(x - 3)}(x - 8)(x + 2)} \qquad \textbf{CUT}$$

$(x \to 3\ ;\ x \neq 3\ \therefore x - 3 \neq 0\ )$

$$= \lim_{x \to 3} \frac{2}{(x - 8)(x + 2)} \qquad \textbf{COPY}$$

$$= \frac{2}{(3 - 8)(3 + 2)} \qquad \textbf{PASTE}$$

$$= \frac{2}{(-5)(5)} = -2 / 25$$

12.

$$\lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{3}{x^2 - x - 2} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{3}{x^2 - 2x + 1x - 2} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{3}{x(x - 2) + 1(x - 2)} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x - 2} - \frac{3}{(x - 2)(x + 1)} \right)$$

$$= \lim_{x \to 2} \frac{x + 1 - 3}{(x - 2)(x + 1)}$$

$$= \lim_{x \to 2} \frac{\cancel{x - 2}}{\cancel{(x - 2)}(x + 1)} \qquad \textbf{CUT}$$

$(x \to 2\ ;\ x \neq 2\ \therefore x - 2 \neq 0\ )$

$$= \lim_{x \to 2} \frac{1}{x + 1} \qquad \textbf{COPY}$$

$$= \frac{1}{2 + 1} \qquad \textbf{PASTE}$$

$$= 1 / 3$$

13.

$$\lim_{x \to 3} \left(\frac{1}{x - 3} - \frac{9x}{x^3 - 27} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{x - 3} - \frac{9x}{x^3 - 3^3} \right)$$

$$= \lim_{x \to 3} \left(\frac{1}{x - 3} - \frac{9x}{(x - 3)(x^2 + 3x + 9)} \right)$$

$$= \lim_{x \to 3} \frac{x^2 + 3x + 9 - 9x}{(x - 3)(x^2 + 3x + 9)}$$

$$= \lim_{x \to 3} \frac{\overbrace{x^2 - 6x + 9}^{-3 \quad -3}}{(x - 3)(x^2 + 3x + 9)}$$

$$= \lim_{x \to 3} \frac{x^2 - 3x - 3x + 9}{(x - 3)(x^2 + 3x + 9)}$$

$$= \lim_{x \to 3} \frac{x(x-3) - 3(x-3)}{(x-3)(x^2+3x+9)}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x^2+3x+9)}$$ **CUT**

$(x \to 3 \; ; \; x \neq 3 \; \therefore x - 3 \neq 0)$

$$= \lim_{x \to 3} \frac{x-3}{x^2+3x+9}$$ **COPY**

$$= \frac{3-3}{3^2+3(3)+9}$$ **PASTE**

$$= 0$$

14.

$$\lim_{x \to 2} \left(\frac{1}{x-2} + \frac{6x}{8-x^3} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x-2} - \frac{6x}{x^3-2^3} \right)$$

$$= \lim_{x \to 2} \left(\frac{1}{x-2} - \frac{6x}{(x-2)(x^2+2x+4)} \right)$$

$$= \lim_{x \to 2} \frac{x^2+2x+4-6x}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x \to 2} \frac{x^2-4x+4}{(x-2)(x^2+2x+4)} \qquad (-2, \; -2)$$

$$= \lim_{x \to 2} \frac{x^2-2x-2x+4}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x \to 2} \frac{x(x-2)-2(x-2)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x^2+2x+4)}$$ **CUT**

$(x \to 2 \; ; \; x \neq 2 \; \therefore x - 2 \neq 0)$

$$= \lim_{x \to 2} \frac{x-2}{x^2+2x+4}$$ **COPY**

$$= \frac{2-2}{2^2+2(2)+4}$$ **PASTE**

$$= 0$$

SOLUTION TO Q SET - 2

01.

$$\lim_{x \to 1} \frac{\sqrt{x+4} - \sqrt{5}}{x-1}$$

$$= \lim_{x \to 1} \frac{\sqrt{x+4} - \sqrt{5}}{x-1} \cdot \frac{\sqrt{x+4} + \sqrt{5}}{\sqrt{x+4} + \sqrt{5}}$$

$$= \lim_{x \to 1} \frac{x+4-5}{x-1} \cdot \frac{1}{\sqrt{x+4} + \sqrt{5}}$$

$$= \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{x-1}} \cdot \frac{1}{\sqrt{x+4} + \sqrt{5}}$$ **CUT**

$(x \to 1 \; ; \; x \neq 1 \; \therefore x - 1 \neq 0)$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x+4} + \sqrt{5}}$$ **COPY**

$$= \frac{1}{\sqrt{1+4} + \sqrt{5}}$$ **PASTE**

$$= \frac{1}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}$$

02.

$$\lim_{x \to 1} \frac{\sqrt{x^2+1}-\sqrt{2}}{x-1}$$

$$= \lim_{x \to 1} \frac{\sqrt{x^2+1}-\sqrt{2}}{x-1} \cdot \frac{\sqrt{x^2+1}+\sqrt{2}}{\sqrt{x^2+1}+\sqrt{2}}$$

$$= \lim_{x \to 1} \frac{x^2+1-2}{x-1} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{2}}$$

$$= \lim_{x \to 1} \frac{x^2-1}{x-1} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{2}}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{2}} \qquad \textbf{CUT}$$

$(x \to 1 \; ; \; x \neq 1 \; \therefore x - 1 \neq 0)$

$$= \lim_{x \to 1} \frac{1}{\sqrt{x^2+1}+\sqrt{2}} \qquad \textbf{COPY}$$

$$= \frac{1}{\sqrt{1+4}+\sqrt{5}} \qquad \textbf{PASTE}$$

$$= \frac{1}{\sqrt{5}+\sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}$$

03.

$$\lim_{x \to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3}$$

$$= \lim_{x \to 3} \frac{\sqrt{x^2+1}-\sqrt{10}}{x-3} \cdot \frac{\sqrt{x^2+1}+\sqrt{10}}{\sqrt{x^2+1}+\sqrt{10}}$$

$$= \lim_{x \to 3} \frac{x^2+1-10}{x-3} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{10}}$$

$$= \lim_{x \to 3} \frac{x^2-9}{x-3} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{10}}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} \cdot \frac{1}{\sqrt{x^2+1}+\sqrt{10}} \qquad \textbf{CUT}$$

$(x \to 3 \; ; \; x \neq 3 \; \therefore x - 3 \neq 0)$

$$= \lim_{x \to 3} \frac{1}{\sqrt{x^2+1}+\sqrt{10}} \qquad \textbf{COPY}$$

$$= \frac{1}{\sqrt{9+1}+\sqrt{10}} \qquad \textbf{PASTE}$$

$$= \frac{1}{\sqrt{10}+\sqrt{10}}$$

$$= \frac{1}{2\sqrt{10}}$$

04.

$$\lim_{x \to 3} \frac{x+1-\sqrt{x+13}}{x-3}$$

$$= \lim_{x \to 3} \frac{x+1-\sqrt{x+13}}{x-3} \cdot \frac{x+1+\sqrt{x+13}}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{(x+1)^2-(x+13)}{x-3} \cdot \frac{1}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{x^2+2x+1-x-13}{x-3} \cdot \frac{1}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{\overset{+4 \qquad -3}{x^2+x-12}}{x-3} \cdot \frac{1}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{x^2+4x-3x-12}{x-3} \cdot \frac{1}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{x(x+4)-3(x+4)}{x-3} \cdot \frac{1}{x+1+\sqrt{x+13}}$$

$$= \lim_{x \to 3} \frac{(x+4)\cancel{(x-3)}}{\cancel{x-3}} \cdot \frac{1}{x+1+\sqrt{x+13}} \qquad \textbf{CUT}$$

$$= \lim_{x \to 3} \frac{x + 4}{x + 1 + \sqrt{x + 13}}$$ **COPY**

$$= \frac{3 + 4}{3 + 1 + \sqrt{3 + 13}}$$ **PASTE**

$$= \frac{7}{4 + 4}$$

$$= 7 / 8$$

05.

$$\lim_{x \to 4} \frac{x^2 + x - 20}{\sqrt{3x + 4} - 4}$$

$$= \lim_{x \to 4} \frac{x^2 + 5x - 4x - 20}{\sqrt{3x + 4} - 4} \; \frac{\sqrt{3x + 4} - 4}{\sqrt{3x + 4} - 4}$$

$$= \lim_{x \to 4} \frac{x(x + 5) - 4(x + 5)}{3x + 4 - 16} \; \frac{\sqrt{3x + 4} + 4}{1}$$

$$= \lim_{x \to 4} \frac{(x + 5)(x - 4)}{3x - 12} \; \frac{\sqrt{3x + 4} + 4}{1}$$

$$= \lim_{x \to 4} \frac{(x + 5)\cancel{(x - 4)}}{3\cancel{(x - 4)}} \; \frac{\sqrt{3x + 4} + 4}{1}$$ **CUT**

$(x \to 4 \; ; \; x \neq 4 \; \therefore x - 4 \neq 0 \;)$

$$= \lim_{x \to 4} \frac{x + 5}{3} \; \frac{\sqrt{3x + 4} + 4}{1}$$ **COPY**

$$= \frac{4 + 5}{3} \; \frac{\sqrt{3(4) + 4} + 4}{1}$$ **PASTE**

$$= \frac{9}{3} \; \frac{\sqrt{12 + 4} + 4}{1}$$

$$= 3 \;\; (4 + 4)$$

$$= 24$$

06.

$$\lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x\sqrt{2(2 + x)}}$$

$$= \lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x\sqrt{2(2 + x)}} \; \frac{\sqrt{2 + x} + \sqrt{2}}{\sqrt{2 + x} + \sqrt{2}}$$

$$= \lim_{x \to 0} \frac{2 + x - 2}{x\sqrt{2(2 + x)}} \; \frac{1}{\sqrt{2 + x} + \sqrt{2}}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}\sqrt{2(2 + x)}} \; \frac{1}{\sqrt{2 + x} + \sqrt{2}}$$ **CUT**

$(x \to 0 \; ; \; x \neq 0 \;)$

$$= \lim_{x \to 0} \frac{1}{\sqrt{2(2 + x)}} \; \frac{1}{\sqrt{2 + x} + \sqrt{2}}$$ **COPY**

$$= \frac{1}{\sqrt{2(2 + 0)}} \; \frac{1}{\sqrt{2 + 0} + \sqrt{2}}$$ **PASTE**

$$= \frac{1}{2} \; \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2} \; \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

07.

$$\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$$

$$= \lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x} \; \frac{\sqrt{1 + x + x^2} + 1}{\sqrt{1 + x + x^2} + 1}$$

$$= \lim_{x \to 0} \frac{1 + x + x^2 - 1}{x} \; \frac{1}{\sqrt{1 + x + x^2} + 1}$$

$$= \lim_{x \to 0} \frac{x + x^2}{x} \; \frac{1}{\sqrt{1 + x + x^2} + 1}$$

$$= \lim_{x \to 0} \frac{\cancel{x}(x+1)}{\cancel{x}} \cdot \frac{1}{\sqrt{1+x+x^2}+1} \qquad \textbf{CUT}$$

$(x \to 0 \; ; \; x \neq 0)$

$$= \lim_{x \to 0} \frac{x+1}{\sqrt{1+x+x^2}+1} \qquad \textbf{COPY}$$

$$= \frac{0+1}{\sqrt{1+0+0}+1} \qquad \textbf{PASTE}$$

$$= \frac{1}{1+1} = 1/2$$

08.

$$\lim_{x \to 3} \frac{x^3-27}{\sqrt{x^2+7}-4}$$

$$= \lim_{x \to 3} \frac{x^3-3^3}{\sqrt{x^2+7}-4} \cdot \frac{\sqrt{x^2+7}+4}{\sqrt{x^2+7}+4}$$

$$= \lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{x^2+7-16} \cdot \frac{\sqrt{x^2+7}+4}{1}$$

$$= \lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{x^2-9} \cdot \frac{\sqrt{x^2+7}+4}{1}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}(x+3)} \cdot \frac{\sqrt{x^2+7}+4}{1}$$

$(x \to 3 \; ; \; x \neq 3 \; \therefore x-3 \neq 0)$

$$= \lim_{x \to 3} \frac{x^2+3x+9}{x+3} \cdot \frac{\sqrt{x^2+7}+4}{1}$$

$$= \frac{3^2+3(3)+9}{3+3} \cdot \frac{\sqrt{3^2+7}+4}{1}$$

$$= \frac{9+9+9}{6} \cdot \frac{\sqrt{9+7}+4}{1}$$

$$= \frac{27}{6} \cdot \frac{4+4}{1}$$

$$= \frac{9}{2}(8) = 36$$

09. $\displaystyle \lim_{x \to 4} \frac{2 - \sqrt{8-x}}{1 - \sqrt{5-x}}$

$$= \lim_{x \to 4} \frac{2 - \sqrt{8-x}}{1 - \sqrt{5-x}} \quad \frac{2 + \sqrt{8-x}}{2 + \sqrt{8-x}} \quad \frac{1 + \sqrt{5-x}}{1 + \sqrt{5-x}}$$

$$= \lim_{x \to 4} \frac{4 - (8-x)}{1 - (5-x)} \quad \frac{1}{2 + \sqrt{8-x}} \quad \frac{1 + \sqrt{5-x}}{1}$$

$$= \lim_{x \to 4} \frac{4 - (8-x)}{1 - (5-x)} \quad \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}}$$

$$= \lim_{x \to 4} \frac{4 - 8 + x}{1 - 5 + x} \quad \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}}$$

$$= \lim_{x \to 4} \frac{x-4}{x-4} \quad \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}}$$

$(x \to 4 \; ; \; x \neq 4 \; \therefore x - 4 \neq 0 \,)$

CUT

$$= \lim_{x \to 4} \frac{1 + \sqrt{5-x}}{2 + \sqrt{8-x}}$$

COPY

$$= \frac{1 + \sqrt{5-4}}{2 + \sqrt{8-4}}$$

PASTE

$$= \frac{1 + \sqrt{1}}{2 + \sqrt{4}}$$

$$= \frac{1+1}{2+2}$$

$$= \frac{1}{2}$$

10. $\displaystyle \lim_{x \to 7} \frac{4 - \sqrt{9+x}}{1 - \sqrt{8-x}}$

$$= \lim_{x \to 7} \frac{4 - \sqrt{9+x}}{1 - \sqrt{8-x}} \quad \frac{4 + \sqrt{9+x}}{4 + \sqrt{9+x}} \quad \frac{1 + \sqrt{8-x}}{1 + \sqrt{8-x}}$$

$$= \lim_{x \to 7} \frac{16 - (9+x)}{1 - (8-x)} \quad \frac{1}{4 + \sqrt{9+x}} \quad \frac{1 + \sqrt{8-x}}{1}$$

$$= \lim_{x \to 7} \frac{16 - 9 - x}{1 - 8 + x} \quad \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}}$$

$$= \lim_{x \to 7} \frac{7 - x}{-7 + x} \quad \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}}$$

$$= \lim_{x \to 7} \frac{-\cancel{(x-7)}}{\cancel{x-7}} \cdot \frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}}$$ **CUT**

$(x \to 7\ ;\ x \neq 7\ \therefore x - 7 \neq 0)$

$$= \lim_{x \to 7} -\frac{1 + \sqrt{8-x}}{4 + \sqrt{9+x}}$$ **COPY**

$$= -\frac{1 + \sqrt{8-7}}{4 + \sqrt{9+7}}$$ **PASTE**

$$= -\frac{1 + \sqrt{1}}{4 + \sqrt{16}}$$

$$= -\frac{1+1}{4+4}$$

$$= -\frac{1}{4}$$

00. $\lim_{x \to 1} \dfrac{\sqrt{x+8} - \sqrt{8x+1}}{\sqrt{5-x} - \sqrt{7x-3}}$ **(EXTRA SUM)**

$$= \lim_{x \to 1} \frac{\sqrt{x+8} - \sqrt{8x+1}}{\sqrt{5-x} - \sqrt{7x-3}} \cdot \frac{\sqrt{x+8} + \sqrt{8x+1}}{\sqrt{x+8} + \sqrt{8x+1}} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{5-x} + \sqrt{7x-3}}$$

$$= \lim_{x \to 1} \frac{x + 8 - (8x+1)}{5 - x - (7x-3)} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \to 1} \frac{x + 8 - 8x - 1}{5 - x - 7x + 3} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \to 1} \frac{-7x + 7}{-8x + 8} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$

$$= \lim_{x \to 1} \frac{-7\cancel{(x-1)}}{-8\cancel{(x-1)}} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$ **CUT**

$(x \to 1\ ;\ x \neq 1\ \therefore x - 1 \neq 0)$

$$= \lim_{x \to 1} \frac{7}{8} \cdot \frac{\sqrt{5-x} + \sqrt{7x-3}}{\sqrt{x+8} + \sqrt{8x+1}}$$ **COPY**

$$= \frac{7}{8} \cdot \frac{\sqrt{5-1} + \sqrt{7-3}}{\sqrt{1+8} + \sqrt{8+1}}$$ **PASTE**

$$= \frac{7}{8} \cdot \frac{2 + 2}{3 + 3}$$

$$= \frac{7}{8} \times \frac{4}{6}$$

$$= 7 / 12$$

11. $$\lim_{x \to 2} \frac{x^2 + \sqrt{x+2} - 6}{x^2 - 4}$$

$$= \lim_{x \to 2} \frac{x^2 - 4 + \sqrt{x+2} - 2}{x^2 - 4}$$

$$= \lim_{x \to 2} \frac{x^2 - 4}{x^2 - 4} + \frac{\sqrt{x+2} - 2}{x^2 - 4}$$

$$= \lim_{x \to 2} 1 + \frac{\sqrt{x+2} - 2}{x^2 - 4} \; \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2}$$

$$= \lim_{x \to 2} 1 + \frac{x + 2 - 4}{x^2 - 4} \; \frac{1}{\sqrt{x+2} + 2}$$

$$= \lim_{x \to 2} 1 + \frac{\cancel{x - 2}}{\cancel{(x - 2)}(x + 2)} \; \frac{1}{\sqrt{x+2} + 2}$$ **CUT**

$(x \to 2 \; ; \; x \neq 2 \; \therefore x - 2 \neq 0)$

$$= \lim_{x \to 2} 1 + \frac{1}{x + 2} \; \frac{1}{\sqrt{x+2} + 2}$$ **COPY**

PASTE

$$= 1 + \frac{1}{2 + 2} \; \frac{1}{\sqrt{2+2} + 2}$$

$$= 1 + \frac{1}{4} \text{ x } \frac{1}{2 + 2}$$

$$= 1 + \frac{1}{16}$$

$$= 17 / 16$$

12. $$\lim_{x \to 3} \frac{x^2 + \sqrt{x+6} - 12}{x^2 - 9}$$

$$= \lim_{x \to 3} \frac{x^2 - 9 + \sqrt{x+6} - 3}{x^2 - 9}$$

$$= \lim_{x \to 3} \frac{x^2 - 9}{x^2 - 9} + \frac{\sqrt{x+6} - 3}{x^2 - 9}$$

$$= \lim_{x \to 3} 1 + \frac{\sqrt{x+6} - 3}{x^2 - 9} \; \frac{\sqrt{x+6} + 3}{\sqrt{x+6} + 3}$$

$$= \lim_{x \to 3} 1 + \frac{x + 6 - 9}{x^2 - 9} \; \frac{1}{\sqrt{x+6} + 3}$$

$$= \lim_{x \to 3} 1 + \frac{\cancel{x - 3}}{\cancel{(x - 3)}(x + 3)} \; \frac{1}{\sqrt{x+6} + 3}$$ **CUT**

$(x \to 3 \; ; \; x \neq 3 \; \therefore x - 3 \neq 0)$

COPY

$$= \lim_{x \to 3} 1 + \frac{1}{x + 3} \; \frac{1}{\sqrt{x+6} + 3}$$

$$= 1 + \frac{1}{3 + 3} \; \frac{1}{\sqrt{3+6} + 3}$$ **PASTE**

$$= 1 + \frac{1}{6} \; \frac{1}{3 + 3}$$

$$= 1 + \frac{1}{36}$$

$$= 37/36$$

13. $$\lim_{x \to 4} \frac{x^2 + \sqrt{x+5} - 19}{x^2 - 16}$$

$$= \lim_{x \to 4} \frac{x^2 - 16 + \sqrt{x+5} - 3}{x^2 - 16}$$

$$= \lim_{x \to 4} \left[\frac{x^2 - 16}{x^2 - 16} + \frac{\sqrt{x+5} - 3}{x^2 - 16} \right]$$

$$= \lim_{x \to 4} \left[1 + \frac{\sqrt{x+5} - 3}{x^2 - 16} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3} \right]$$

$$= \lim_{x \to 4} \left[1 + \frac{x + 5 - 9}{x^2 - 16} \cdot \frac{1}{\sqrt{x+5} + 3} \right]$$

$$= \lim_{x \to 4} \left[1 + \frac{\cancel{x - 4}}{\cancel{(x - 4)}(x + 4)} \cdot \frac{1}{\sqrt{x+5} + 3} \right]$$ **CUT**

$(x \to 4 \; ; \; x \neq 4 \; \therefore x - 4 \neq 0)$

COPY

$$= \lim_{x \to 4} \left[1 + \frac{1}{x + 4} \cdot \frac{1}{\sqrt{x+5} + 3} \right]$$

$$= 1 + \frac{1}{4 + 4} \cdot \frac{1}{\sqrt{4+5} + 3}$$ **PASTE**

$$= 1 + \frac{1}{8} \cdot \frac{1}{3 + 3}$$

$$= 1 + \frac{1}{48}$$

$$= 49/48$$

SOLUTION TO Q SET - 3

01. $$\lim_{x \to 2} \frac{x^3 + 3x^2 - 9x - 2}{x^3 - x - 6}$$

$x^3 + 3x^2 - 9x - 2 = (x - 2)(\; ? \;)$

2	1	3	− 9	−2
		2	10	2
	1	5	1	0

$x^3 + 3x^2 - 9x - 2$

$= (x - 2)(x^2 + 5x + 1)$

$x^3 - x - 6 = (x - 2)(\; ? \;)$

2	1	0	− 1	−6
		2	4	6
	1	2	3	0

$x^3 - x - 6 = (x - 2)(x^2 + 2x + 3)$

BACK INTO THE SUM

$$= \lim_{x \to 2} \frac{\cancel{(x - 2)}(x^2 + 5x + 1)}{\cancel{(x - 2)}(x^2 + 2x + 3)}$$ **CUT**

$(x \to 2 \; ; \; x \neq 2 \; \therefore x - 2 \neq 0)$

$$= \lim_{x \to 2} \frac{x^2 + 5x + 1}{x^2 + 2x + 3}$$ **COPY**

$$= \frac{2^2 + 5(2) + 1}{2^2 + 2(2) + 3}$$ **PASTE**

$$= \frac{4 + 10 + 1}{4 + 4 + 3}$$

02. $\lim\limits_{x \to 3} \dfrac{x^3 - x - 24}{x^3 + x^2 - 36}$

$x^3 - x - 24 = (x - 3)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & 0 & -1 & -24 \\ & & 3 & 9 & 24 \\ \hline & 1 & 3 & 8 & 0 \end{array}$$

$x^3 - x - 24 = (x - 3)(x^2 + 3x + 8)$

$x^3 + x^2 - 36 = (x - 2)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & 1 & 0 & -36 \\ & & 3 & 12 & 36 \\ \hline & 1 & 4 & 12 & 0 \end{array}$$

$x^3 + x^2 - 36 = (x - 3)(x^2 + 4x + 12)$

BACK INTO THE SUM

$= \lim\limits_{x \to 3} \dfrac{\cancel{(x - 3)}(x^2 + 3x + 8)}{\cancel{(x - 3)}(x^2 + 4x + 12)}$ **CUT**

$(x \to 3\ ;\ x \neq 3\ \therefore x - 3 \neq 0\ )$

$= \lim\limits_{x \to 3} \dfrac{x^2 + 3x + 8}{x^2 + 4x + 12}$ **COPY**

$= \dfrac{3^2 + 3(3) + 8}{3^2 + 4(3) + 12}$ **PASTE**

$= \dfrac{9 + 9 + 8}{9 + 12 + 12}$

$= \dfrac{26}{33}$

03. $\lim\limits_{x \to 3} \dfrac{x^3 - 4x - 15}{x^3 + x^2 - 6x - 18}$

$x^3 - 4x - 15 = (x - 3)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & 0 & -4 & -15 \\ & & 3 & 9 & 15 \\ \hline & 1 & 3 & 5 & 0 \end{array}$$

$x^3 - x - 24 = (x - 3)(x^2 + 3x + 5)$

$x^3 + x^2 - 6x - 18 = (x - 2)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & 1 & -6 & -18 \\ & & 3 & 12 & 18 \\ \hline & 1 & 4 & 6 & 0 \end{array}$$

$x^3 + x^2 - 6x - 18$

$= (x - 2)(x^2 + 4x + 6)$

BACK INTO THE SUM

$= \lim\limits_{x \to 3} \dfrac{\cancel{(x - 3)}(x^2 + 3x + 5)}{\cancel{(x - 3)}(x^2 + 4x + 6)}$ **CUT**

$(x \to 3\ ;\ x \neq 3\ \therefore x - 3 \neq 0\ )$

$= \lim\limits_{x \to 3} \dfrac{x^2 + 3x + 5}{x^2 + 4x + 6}$ **COPY**

$= \dfrac{3^2 + 3(3) + 5}{3^2 + 4(3) + 6}$ **PASTE**

$= \dfrac{9 + 9 + 5}{9 + 12 + 6}$

$= \dfrac{23}{27}$

04. $\lim\limits_{x \to 1} \dfrac{3x^3 + 4x^2 - 6x - 1}{2x^3 - x - 1}$

$3x^3 + 4x^2 - 6x - 1 = (x - 1)\ (\ ?\ )$

$$\begin{array}{c|cccc} 1 & 3 & 4 & -6 & -1 \\ & & 3 & 7 & 1 \\ \hline & 3 & 7 & 1 & 0 \end{array}$$

$3x^3 + 4x^2 - 6x - 1$

$= (x - 1)\ (3x^2 + 7x + 1)$

$2x^3 - x - 1 = (x - 1)\ (\ ?\ )$

$$\begin{array}{c|cccc} 1 & 2 & 0 & -1 & -1 \\ & & 2 & 2 & 1 \\ \hline & 2 & 2 & 1 & 0 \end{array}$$

$2x^3 - x - 1 = (x - 1)\ (2x^2 + 2x + 1)$

BACK INTO THE SUM

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x - 1)}\ (3x^2 + 7x + 1)}{\cancel{(x - 1)}\ (2x^2 + 2x + 1)}$ **CUT**

$(x \to 1\ ;\ x \neq 1\ \therefore x - 1 \neq 0\ )$

$= \lim\limits_{x \to 1} \dfrac{3x^2 + 7x + 1}{2x^2 + 2x + 1}$ **COPY**

$= \dfrac{3(1)^2 + 7(1) + 1}{2(1)^2 + 2(1) + 1}$ **PASTE**

$= \dfrac{3 + 7 + 1}{2 + 2 + 1}$

$= \dfrac{11}{5}$

05. $\lim\limits_{x \to 2} \dfrac{x^3 - 3x^2 + 4}{x^3 - 2x^2 - 4x + 8}$

$x^3 - 3x^2 + 4 = (x - 2)\ (\ ?\ )$

$$\begin{array}{c|cccc} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$x^3 - 3x^2 + 4 = (x - 2)\ (x^2 - x - 2)$

$x^3 - 2x^2 - 4x + 8 = (x - 2)\ (\ ?\ )$

$$\begin{array}{c|cccc} 2 & 1 & -2 & -4 & 8 \\ & & 2 & 0 & -8 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$x^3 - 2x^2 - 4x + 8 = (x - 2)\ (x^2 - 4)$

BACK INTO THE SUM

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x - 2)}\ (x^2 - x - 2)}{\cancel{(x - 2)}\ (x^2 - 4)}$ **CUT**

$(x \to 2\ ;\ x \neq 2\ \therefore x - 2 \neq 0\ )$

$= \lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 4}$ **COPY**

$= \lim\limits_{x \to 2} \dfrac{x^2 - 2x + x - 2}{(x - 2)(x + 2)}$

$= \lim\limits_{x \to 2} \dfrac{x(x - 2) + 1(x - 2)}{(x - 2)(x + 2)}$

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x - 2)}(x + 1)}{\cancel{(x - 2)}(x + 2)}$ **CUT**

$= \lim\limits_{x \to 2} \dfrac{x + 1}{x + 2}$ **COPY**

$= \dfrac{2 + 1}{2 + 2}$ **PASTE**

$= 3\ /\ 4$

06. $\lim\limits_{x \to 1} \dfrac{x^3 - x^2 - x + 1}{x^3 + x^2 - 5x + 3}$

$x^3 - x^2 - x + 1 = (x - 1)(\ ?\ )$

$$\begin{array}{c|cccc} 1 & 1 & -1 & -1 & 1 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$x^3 - x^2 - x + 1 = (x - 1)(x^2 - 1)$

$x^3 + x^2 - 5x + 3 = (x - 1)(\ ?\ )$

$$\begin{array}{c|cccc} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^3 + x^2 - 5x + 3$

$= (x - 1)(x^2 + 2x - 3)$

BACK INTO THE SUM

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x - 1)}(x^2 - 1)}{\cancel{(x - 1)}(x^2 + 2x - 3)}$ **CUT**

$(x \to 1\ ;\ x \neq 1\ \therefore x - 1 \neq 0)$

$= \lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 + 2x - 3}$ **COPY**

$= \lim\limits_{x \to 1} \dfrac{(x - 1)(x + 1)}{x^2 + 3x - x - 3}$

$= \lim\limits_{x \to 1} \dfrac{(x - 1)(x + 1)}{x(x + 3) - 1(x + 3)}$

$= \lim\limits_{x \to 1} \dfrac{\cancel{(x - 1)}(x + 1)}{\cancel{(x - 1)}(x + 3)}$ **CUT**

$= \lim\limits_{x \to 1} \dfrac{x + 1}{x + 3}$ **COPY**

$= \dfrac{1 + 1}{1 + 3} = \dfrac{1}{2}$

07. $\lim\limits_{x \to 3} \dfrac{x^3 - 3x^2 - 9x + 27}{x^3 - 6x^2 - 9x}$

$x^3 - 3x^2 - 9x + 27 = (x - 3)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & -3 & -9 & 27 \\ & & 3 & 0 & -27 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$x^3 - 3x^2 - 9x + 27 = (x - 3)(x^2 - 9)$

$x^3 - 6x^2 - 9x = (x - 3)(\ ?\ )$

$$\begin{array}{c|cccc} 3 & 1 & -6 & 9 & 0 \\ & & 3 & -9 & 0 \\ \hline & 1 & -3 & 0 & 0 \end{array}$$

$x^3 - 6x^2 - 9x = (x - 3)(x^2 - 3x)$

BACK INTO THE SUM

$= \lim\limits_{x \to 3} \dfrac{\cancel{(x - 3)}(x^2 - 9)}{\cancel{(x - 3)}(x^2 - 3x)}$ **CUT**

$(x \to 3\ ;\ x \neq 3\ \therefore x - 3 \neq 0)$

$= \lim\limits_{x \to 3} \dfrac{x^2 - 9}{x^2 - 3x}$ **COPY**

$= \lim\limits_{x \to 3} \dfrac{(x - 3)(x + 3)}{x(x - 3)}$ **CUT**

$= \lim\limits_{x \to 3} \dfrac{x + 3}{x}$ **COPY**

$= \dfrac{3 + 3}{3}$ **PASTE**

$= 2$

SOLUTION TO Q SET - 4

01. $\lim_{x\to a} \dfrac{x^{25} - a^{25}}{x^{15} - a^{15}}$

Divide numerator and denominator by x – a

$x \to a \; ; x \neq a \quad \therefore x - a \neq 0$

$$= \lim_{x\to a} \frac{\dfrac{x^{25} - a^{25}}{x - a}}{\dfrac{x^{15} - a^{15}}{x - a}}$$

$$= \frac{25\, a^{25-1}}{15\, a^{15-1}}$$

$$= \frac{25(a)^{24}}{15(a)^{14}}$$

$$= \frac{5(a)^{24-14}}{3}$$

$$= \frac{5a^{10}}{3}$$

02. $\lim_{x\to a} \dfrac{x^{7} - a^{7}}{x^{11} - a^{11}}$

Divide numerator and denominator by x – a

$$= \lim_{x\to a} \frac{\dfrac{x^{7} - a^{7}}{x - a}}{\dfrac{x^{11} - a^{11}}{x - a}}$$

$$= \frac{7\, a^{7-1}}{11\, a^{11-1}}$$

$$= \frac{7\, a^{6}}{11\, a^{10}}$$

$$= \frac{7}{11a^{4}}$$

03. $\lim_{x\to 4} \dfrac{x^{3} - 64}{x^{2} - 16}$

$$= \lim_{x\to 4} \frac{x^{3} - 4^{3}}{x^{2} - 4^{2}}$$

Divide numerator and denominator by $x - 4$, $x \to 4 \; ; x \neq 4 \quad \therefore x - 4 \neq 0$

$$= \lim_{x\to 4} \frac{\dfrac{x^{3} - 4^{3}}{x - 4}}{\dfrac{x^{2} - 4^{2}}{x - 4}}$$

$$= \frac{3\,.\,(4)^{3-1}}{2\,.\,(4)^{2-1}}$$

$$= \frac{3\,.\,4^{2}}{2\,.\,4^{1}}$$

$$= \frac{3\,.\,4}{2}$$

$$= 6$$

04. $\lim_{x\to 2} \dfrac{x^{6} - 64}{x^{10} - 1024}$

$$= \lim_{x\to 2} \frac{x^{6} - 2^{6}}{x^{10} - 2^{10}}$$

Divide numerator and denominator by x – 2

$x \to 2 \; ; x \neq 2 \quad \therefore x - 2 \neq 0$

$$= \lim_{x\to 2} \frac{\dfrac{x^{6} - 2^{6}}{x - 2}}{\dfrac{x^{10} - 2^{10}}{x - 2}}$$

$$= \frac{6\,.\,(2)^{6-1}}{10\,.\,(2)^{10-1}}$$

$$= \frac{6 \,.\, 2^5}{10 \,.\, 2^9}$$

$$= \frac{6}{10 .\, 2^4}$$

$$= \frac{6}{10 \,.\, 16}$$

$$= \frac{3}{80}$$

05. $\lim\limits_{x \to a} \dfrac{x^{-3} - a^{-3}}{x^{-7} - a^{-7}}$

Divide numerator and denominator by x – a

$x \to a$; $x \neq a$ $\therefore$ $x - a \neq 0$

$$= \lim_{x \to a} \frac{\dfrac{x^{-3} - a^{-3}}{x - a}}{\dfrac{x^{-7} - a^{-7}}{x - a}}$$

$$= \frac{-3\ a^{-3-1}}{-7\ a^{-7-1}}$$

$$= \frac{3\ a^{-4}}{7\ a^{-8}}$$

$$= \frac{3}{7}\ a^{-4+8}$$

$$= \frac{3\ a^4}{7}$$

06. $\lim\limits_{x \to 3} \dfrac{x^{-4} - 3^{-4}}{x^{-7} - 3^{-7}}$

Divide numerator and denominator by x – 3

$x \to 3$; $x \neq 3$ $\therefore$ $x - 3 \neq 0$

$$= \lim_{x \to 3} \frac{\dfrac{x^{-4} - 3^{-4}}{x - 3}}{\dfrac{x^{-7} - 3^{-7}}{x - 3}}$$

$$= \frac{-4\ .3^{-4-1}}{-7\ .3^{-7-1}}$$

$$= \frac{4}{7}\ \frac{3^{-5}}{3^{-8}}$$

$$= \frac{4}{7}\ .\ 3^{-5+8}$$

$$= \frac{4}{7}\ (3)^3$$

$$= \frac{108}{7}$$

07. $\lim\limits_{x \to 3} \dfrac{x^{1/4} - 3^{1/4}}{x^{1/3} - 3^{1/3}}$

Divide numerator and denominator by x – 3

$x \to 3$; $x \neq 3$ $\therefore$ $x - 3 \neq 0$

$$= \lim_{x \to 3} \frac{\dfrac{x^{1/4} - 3^{1/4}}{x - 3}}{\dfrac{x^{1/3} - 3^{1/3}}{x - 3}}$$

$$= \frac{\frac{1}{4}\ .\ 3^{1/4 - 1}}{\frac{1}{3}\ .\ 3^{1/3 - 1}}$$

$$= \frac{3}{4}\ 3^{1/4 - 1 - 1/3 + 1}$$

$$= \frac{3}{4}\ .\ 3^{1/4\ -\ 1/3}$$

$$= \frac{3\,(3)^{-1/12}}{4}$$

$$= \frac{3^{\,1-1/12}}{4}$$

$$= \frac{3^{\,-11/12}}{4}$$

08. $\lim\limits_{x\to k} \dfrac{x^8 - k^8}{x - k} = 8$, find k

$8k^{(8-1)} = 8$

$8k^7 = 8$

$k^7 = 1$

$k = 1$

09. $\lim\limits_{x\to k} \dfrac{x^5 - k^5}{x - k} = 80$, find k

$5k^{(5-1)} = 80$

$5k^4 = 80$

$k^4 = 16$

$k^2 = 4$

$k = \pm 2$

10. $\lim\limits_{x\to 1} \dfrac{x + x^2 + x^3 - 3}{x - 1}$

Solution

$$= \lim_{x\to 1} \frac{x + x^2 + x^3 - 3}{x - 1}$$

$$= \lim_{x\to 1} \frac{x - 1 + x^2 - 1 + x^3 - 1}{x - 1}$$

$$= \lim_{x\to 1} \frac{x - 1 + x^2 - 1^2 + x^3 - 1^3}{x - 1}$$

$$= \lim_{x\to 1} \frac{x - 1}{x - 1} + \frac{x^2 - 1^2}{x - 1} + \frac{x^3 - 1^3}{x - 1}$$

$$= 1 + 2(1)^{2-1} + 3(1)^{3-1}$$

$$= 1 + 2(1)^1 + 3(1)^2$$

$$= 1 + 2(1) + 3(1)$$

$$= 6$$

11. $\lim\limits_{x\to 3} \dfrac{x + x^2 + x^3 - 39}{x - 3}$

$$= \lim_{x\to 3} \frac{x + x^2 + x^3 - 39}{x - 3}$$

$$= \lim_{x\to 3} \frac{x - 3 + x^2 - 9 + x^3 - 27}{x - 3}$$

$$= \lim_{x\to 3} \frac{x - 3 + x^2 - 3^2 + x^3 - 3^3}{x - 3}$$

$$= \lim_{x\to 3} \frac{x - 3}{x - 3} + \frac{x^2 - 3^2}{x - 3} + \frac{x^3 - 3^3}{x - 3}$$

$$= 1 + 2(3)^{2-1} + 3(3)^{3-1}$$

$$= 1 + 2(3)^1 + 3(3)^2$$

$$= 1 + 2(3) + 3(9)$$

$$= 1 + 6 + 27$$

$$= 34$$

SOLUTION TO Q SET - 5

01. Discuss whether the limit exist as $x \to 3$

$$f(x) = \begin{cases} x^2 + x + 1, & 2 \le x \le 3 \\ 2x + 1, & 3 < x \le 4 \end{cases}$$

✓ $\lim_{x \to 3-} f(x)$

$= \lim_{x \to 3-} x^2 + x + 1$

$= 3^2 + 3 + 1 = 13$

✓ $\lim_{x \to 3+} f(x)$

$= \lim_{x \to 3+} 2x + 1$

$= 6 + 1 = 7$

Since $\lim_{x \to 3-} f(x) \neq \lim_{x \to 3+} f(x)$

$\lim_{x \to 3} f(x)$ does not exists

02. Discuss whether the limit exist as $x \to 3$

$$f(x) = \begin{cases} x^2 - 3x + 7, & x \le 3 \\ x + 1, & 3 < x \end{cases}$$

✓ $\lim_{x \to 3-} f(x)$

$= \lim_{x \to 3-} x^2 - 3x + 7$

$= 3^2 - 9 + 7 = 7$

✓ $\lim_{x \to 3+} f(x)$

$= \lim_{x \to 3+} x + 1$

$= 3 + 1 = 4$

Since $\lim_{x \to 3-} f(x) \neq \lim_{x \to 3+} f(x)$

$\lim_{x \to 3} f(x)$ does not exists

03. Discuss whether the limit exist as $x \to 2$

$$f(x) = \begin{cases} 4x + 3, & x \le 2 \\ 2x^2 + 3, & x > 2 \end{cases}$$

✓ $\lim_{x \to 2-} f(x)$

$= \lim_{x \to 2-} 4x + 3$

$= 8 + 3 = 11$

✓ $\lim_{x \to 2+} f(x)$

$= \lim_{x \to 2+} 2x^2 + 3$

$= 8 + 3 = 11$

Since $\lim_{x \to 2-} f(x) = \lim_{x \to 2+} f(x)$

$\lim_{x \to 2} f(x)$ does exists

04. Discuss whether the limit exist as $x \to 0$

$$f(x) = \begin{cases} x^2 + 1, & 0 \le x \le 2 \\ 2\sqrt{x^2 + 1} - 1, & -2 \le x < 0 \end{cases}$$

✓ $\lim_{x \to 0-} f(x)$

$= \lim_{x \to 0-} 2\sqrt{x^2 + 1} - 1$

$= 2(1) - 1 = 1$

✓ $\lim_{x \to 0+} f(x)$

$= \lim_{x \to 0+} x^2 + 1$

$= 0 + 1 = 1$

Since $\lim_{x \to 0-} f(x) = \lim_{x \to 0+} f(x)$

$\lim_{x \to 0} f(x)$ does exists

05. Discuss whether the limit exist as $x \to 1$

$$f(x) = 5x - 1 \quad , \quad x \leq 1$$
$$= \frac{2x^2 - 1}{x + 5} \quad , \quad x > 1$$

✓ $\lim\limits_{x \to 1^-} f(x)$

$$= \lim_{x \to 1^-} 5x - 1$$

$$= 5 - 1 = 4$$

✓ $\lim\limits_{x \to 1^+} f(x)$

$$= \lim_{x \to 1^+} \frac{2x^2 - 1}{x + 5}$$

$$= \frac{2 - 1}{1 + 5} = \frac{1}{6}$$

Since $\lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$

$\lim\limits_{x \to 1} f(x)$ does not exists

Q SET - 1

01 . $\lim_{x \to 0} \frac{\sin x \cdot \sin 3x}{2x^2}$ ans : $3/2$

02. $\lim_{x \to 0} \frac{\sin 3x \cdot \sin 5x}{8x^2}$ ans : $15/8$

03. $\lim_{x \to 0} \frac{\sin 3x - \sin 2x}{x}$ ans : 1

04. $\lim_{x \to 0} \frac{\sin 6x - \sin 4x}{x}$ ans : 2

05. $\lim_{x \to 0} \frac{5\sin x - x.\cos x}{2\tan x + x^2}$ ans : 2

06. $\lim_{x \to 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$ ans : 2

07. $\lim_{x \to 0} \frac{7x \cos x + 3 \sin x}{3x^2 + \tan x}$ ans : 10

08. $\lim_{x \to 0} \frac{4 \sin x - 3 \tan x}{2x + 3 \sin x}$ ans : $1/5$

09. $\lim_{x \to 0} \frac{\cos 3x - \cos 7x}{x^2}$ ans : 20

10. $\lim_{x \to 0} \frac{\cos 5x - \cos 11x}{x^2}$ ans : 48

11. $\lim_{x \to 0} \frac{x^2}{\cos 3x - \cos 9x}$ ans : $1/36$

12. $\lim_{x \to 0} \frac{\cos 4x - \cos 8x}{x.\tan x}$ ans : 24

13. $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$ ans : 2

14. $\lim_{x \to 0} \frac{\sin x \cdot (1 - \cos x)}{x^3}$ ans : $1/2$

15. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$ ans : $1/2$

Q SET - 2

01. $\lim_{x \to \pi/2} \frac{\cot^2 x}{1 - \sin x}$ ans : 1

02. $\lim_{x \to 0} \frac{\tan^2 x}{1 - \cos x}$ ans : 2

03. $\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2}$ ans : –8

04. $\lim_{x \to \pi/2} \frac{\cos^2 x}{\sqrt{3 + \sin x} - 2}$ ans : –8

05. $\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ ans : $1/4\sqrt{2}$

06. $\lim\limits_{x \to \pi} \dfrac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x}$ ans : $1/4\sqrt{2}$

07. $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{1 - \cos^3 x}$ ans : $^2/_3$

08. $\lim\limits_{x \to \pi} \dfrac{\sin^2 x}{1 + \cos^3 x}$ ans : $^2/_3$

09. $\lim\limits_{x \to \pi/2} \dfrac{1 - \sin^3 x}{\cos^2 x}$ ans : $^3/_2$

10. $\lim\limits_{x \to \pi/4} \dfrac{1 - \tan^3 x}{\sec^2 x - 2}$ ans : $^{-3}/_2$

11. $\lim\limits_{x \to \pi/4} \dfrac{\text{cosec}^2 x - 2}{1 - \cot^3 x}$ ans : $^{-2}/_3$

12. $\lim\limits_{x \to \pi/4} \dfrac{\cot x - 1}{2 - \text{cosec}^2 x}$ ans : $^{-1}/_2$

14. $\lim\limits_{x \to \pi/6} \dfrac{2 - \text{cosec}\, x}{\cot^2 x - 3}$ ans : $^{-1}/_4$

15. $\lim\limits_{x \to \pi/3} \dfrac{\sec^3 x - 8}{\tan^2 x - 3}$ ans : 3

Q SET - 3

01. $\lim\limits_{x \to \pi} \dfrac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$ ans : $^1/_4$

02. $\lim\limits_{x \to \pi} \dfrac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2}$ ans : $^1/_8$

03. $\lim\limits_{x \to \pi/4} \dfrac{1 - \tan x}{\pi - 4x}$ ans : $^1/_2$

04. $\lim\limits_{x \to \pi/3} \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ ans : $^4/_3$

05. $\lim\limits_{x \to \pi/4} \dfrac{\cos x - \sin x}{\pi - 4x}$ ans : $^{\sqrt{2}}/_4$

06. $\lim\limits_{x \to 1} \dfrac{1 + \cos \pi x}{(1 - x)^2}$ ans : $\pi^2/2$

07. $\lim\limits_{x \to \pi/2} \dfrac{\text{cosec}\, x - 1}{(\pi/2 - x)^2}$ ans : $^1/_2$

08. $\lim\limits_{x \to \pi/2} \dfrac{3\cos x + \cos 3x}{(\pi - 2x)^3}$ ans : $^1/_2$

09. $\lim\limits_{x \to \pi/2} \dfrac{\sec x - \tan x}{\pi/2 - x}$ ans : $^1/_2$

SOLUTION TO Q SET - 1

01.

$$\lim_{x \to 0} \frac{\sin x \cdot \sin 3x}{2x^2}$$

$$= \lim_{x \to 0} \frac{1}{2} \frac{\sin x \cdot \sin 3x}{x^2}$$ **SIDE**

$$= \lim_{x \to 0} \frac{1}{2} \frac{\sin x}{x} \cdot \frac{\sin 3x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} \frac{1}{2} \frac{\sin x}{x} \cdot 3 \frac{\sin 3x}{3x}$$

$$= \frac{1}{2} (1) \cdot 3 \cdot (1)$$ **KAHI FORMULA** **KAHI PASTE**

$$= \frac{3}{2}$$

02.

$$\lim_{x \to 0} \frac{\sin 3x \cdot \sin 5x}{8x^2}$$

$$= \lim_{x \to 0} \frac{1}{8} \frac{\sin 3x \cdot \sin 5x}{x^2}$$ **SIDE**

$$= \lim_{x \to 0} \frac{1}{2} \frac{\sin 3x}{x} \cdot \frac{\sin 5x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} \frac{1}{2}\, 3 \frac{\sin 3x}{3x} \cdot 5 \frac{\sin 5x}{5x}$$

$$= \frac{1}{2} 3(1) \cdot 5 \cdot (1)$$ **KAHI FORMULA** **KAHI PASTE**

$$= \frac{15}{2}$$

03.

$$\lim_{x \to 0} \frac{\sin 3x - \sin 2x}{x}$$

$$= \lim_{x \to 0} \frac{\sin 3x}{x} - \frac{\sin 2x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} 3 \frac{\sin 3x}{3x} - 2 \frac{\sin 2x}{2x}$$

$$= 3(1) - 2 \cdot (1)$$ **KAHI FORMULA** **KAHI PASTE**

$$= 1$$

04.

$$\lim_{x \to 0} \frac{\sin 6x - \sin 4x}{x}$$

$$= \lim_{x \to 0} \frac{\sin 6x}{x} - \frac{\sin 4x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} 6 \frac{\sin 6x}{6x} - 4 \frac{\sin 4x}{4x}$$

$$= 6(1) - 4 \cdot (1)$$ **KAHI FORMULA** **KAHI PASTE**

$$= 2$$

05.

$$\lim_{x \to 0} \frac{5\sin x - x.\cos x}{2\tan x + x^2}$$

Divide Numerator & Denominator by x ,

$x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\dfrac{5\sin x - x.\cos x}{x}}{\dfrac{2\tan x + x^2}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{5 \sin x}{x} - \frac{\cancel{x}.\cos x}{\cancel{x}}}{\frac{2\tan x}{x} + \frac{x^{\cancel{2}}}{\cancel{x}}}$$

DISITRIBUTE

CUT

$$= \lim_{x \to 0} \frac{\frac{5 \sin x}{x} - \cos x}{2\frac{\tan x}{x} + x}$$

COPY

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= \frac{5(1) - \cos 0}{2(1) + 0}$$

$$= \frac{5 - 1}{2}$$

$$= 2$$

06.

$$\lim_{x \to 0} \frac{x \cos x + \sin x}{x^2 + \tan x}$$

Divide Numerator & Denominator by x , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{x \cos x + \sin x}{x}}{\frac{x^2 + \tan x}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{\cancel{x} \cos x}{\cancel{x}} + \frac{\sin x}{x}}{\frac{x^{\cancel{2}}}{\cancel{x}} + \frac{\tan x}{x}}$$

DISITRIBUTE

CUT

$$= \lim_{x \to 0} \frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}}$$

COPY

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= \frac{\cos 0 + 1}{0 + 1}$$

$$= 1 + 1$$

$$= 2$$

07.

$$\lim_{x \to 0} \frac{7x \cos x + 3 \sin x}{3x^2 + \tan x}$$

Divide Numerator & Denominator by x , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{7x \cos x + 3 \sin x}{x}}{\frac{3x^2 + \tan x}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{7\cancel{x} \cos x}{\cancel{x}} + \frac{3 \sin x}{x}}{\frac{3x^{\cancel{2}}}{\cancel{x}} + \frac{\tan x}{x}}$$

DISITRIBUTE

CUT

$$= \lim_{x \to 0} \frac{7 \cos x + 3 \frac{\sin x}{x}}{3x + \frac{\tan x}{x}}$$

COPY

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= \frac{7\cos 0 + 3(1)}{3(0) + 1}$$

$$= 7 + 3$$

$$= 10$$

08.

$$\lim_{x \to 0} \frac{4 \sin x - 3 \tan x}{2x + 3 \sin x}$$

Divide Numerator & Denominator by x ,

$x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{4 \sin x - 3 \tan x}{x}}{\frac{2x + 3 \sin x}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{4 \sin x}{x} - \frac{3 \tan x}{x}}{\cancel{\frac{2x}{x}} + \frac{3 \sin x}{x}}$$ **DISITRIBUTE** **CUT**

$$= \lim_{x \to 0} \frac{4 \frac{\sin x}{x} - 3 \frac{\tan x}{x}}{2 + 3 \frac{\sin x}{x}}$$ **COPY**

LIMIT JAThI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= \frac{4(1) - 3(1)}{2 + 3(1)}$$

$= 1/5$

$= 10$

09.

$$\lim_{x \to 0} \frac{\cos 3x - \cos 7x}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \sin\left(\frac{3x + 7x}{2}\right) . \sin\left(\frac{3x - 7x}{2}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \sin 5x . \sin(-2x)}{x^2}$$

$$= \lim_{x \to 0} \frac{2 . \sin 5x . \sin 2x}{x^2}$$

$$= \lim_{x \to 0} 2 . \frac{\sin 5x}{x} . \frac{\sin 2x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} 2 . 5 \frac{\sin 5x}{5x} . 2 \frac{\sin 2x}{2x}$$

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$= 2.5(1) . 2(1)$

$= 20$

10.

$$\lim_{x \to 0} \frac{\cos 5x - \cos 11x}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \sin\left(\frac{5x+11x}{2}\right) . \sin\left(\frac{5x-11x}{2}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \sin 8x . \sin(-3x)}{x^2}$$

$$= \lim_{x \to 0} \frac{2 . \sin 8x . \sin 3x}{x^2}$$

$$= \lim_{x \to 0} 2 . \frac{\sin 8x}{x} . \frac{\sin 3x}{x}$$ **DISITRIBUTE**

$$= \lim_{x \to 0} 2 . 8 \frac{\sin 8x}{8x} . 3 \frac{\sin 3x}{3x}$$

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$= 2.8(1) . 3(1) = 48$

11. $\lim\limits_{x \to 0} \dfrac{x^2}{\cos 3x - \cos 9x}$

$= \lim\limits_{x \to 0} \dfrac{x^2}{-2\sin\left(\dfrac{3x+9x}{2}\right) . \sin\left(\dfrac{3x-9x}{2}\right)}$

$= \lim\limits_{x \to 0} \dfrac{x^2}{-2\sin 6x . \sin(-3x)}$

$= \lim\limits_{x \to 0} \dfrac{x^2}{2\sin 6x . \sin 3x}$

$= \lim\limits_{x \to 0} \dfrac{x}{2\sin 6x} \dfrac{x}{. \sin 3x}$

$= \lim\limits_{x \to 0} \dfrac{1}{2} \dfrac{1}{6} \dfrac{6x}{\sin 6x} . \dfrac{1}{3} \dfrac{3x}{\sin 3x}$

$= \dfrac{1}{2} . \dfrac{1}{6} . (1) \dfrac{1}{3} (1)$

$= \dfrac{1}{36}$

12. $\lim\limits_{x \to 0} \dfrac{\cos 4x - \cos 8x}{x.\tan x}$

$= \lim\limits_{x \to 0} \dfrac{-2\sin\left(\dfrac{4x+8x}{2}\right) . \sin\left(\dfrac{4x-8x}{2}\right)}{x.\tan x}$

$= \lim\limits_{x \to 0} \dfrac{-2\sin 6x . \sin(-2x)}{x.\tan x}$

$= \lim\limits_{x \to 0} \dfrac{2 . \sin 6x . \sin 2x}{x.\tan x}$

Divide Numerator & Denominator by x^2 ,

$x \to 0$, $x \neq 0$, $x^2 \neq 0$

$= \lim\limits_{x \to 0} \dfrac{\dfrac{2 . \sin 6x . \sin 2x}{x^2}}{\dfrac{x \tan x}{x^2}}$

$= \lim\limits_{x \to 0} \dfrac{2 . \dfrac{\sin 6x}{x} . \dfrac{\sin 2x}{x}}{\dfrac{\cancel{x} \tan x}{\cancel{x^2}}}$ **DISITRIBUTE**

CUT

$= \lim\limits_{x \to 0} \dfrac{2 . 6 \dfrac{\sin 6x}{6x} . 2 \dfrac{\sin 2x}{2x}}{\dfrac{\tan x}{x}}$

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$= \dfrac{2.6(1) . 2(1)}{1} = 24$

13. $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

$= \lim\limits_{x \to 0} \dfrac{2. \sin^2 x}{x^2}$

$= \lim\limits_{x \to 0} 2\left(\dfrac{\sin x}{x}\right)^2$ SQUARE–SQURE THE WHOLE SQUARE

$= 2(1)^2$

$= 2$

14.

$$\lim_{x \to 0} \frac{\sin x \,.\, (1 - \cos x)}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin x \,.\, 2\sin^2 (x/2)}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \,.\, \frac{2\sin^2 (x/2)}{x^2}$$ DISTRIBUTE

SQUARE–SQUARE THE WHOLE SQUARE

$$= \lim_{x \to 0} \frac{\sin x}{x} \,.\, 2\left(\frac{\sin (x/2)}{x}\right)^2$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \,.\, 2\left(\frac{1}{2}\frac{\sin (x/2)}{x/2}\right)^2$$

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= 1 \;.\; 2\left(\frac{1}{2}\,(1)\right)^2 = \frac{1}{2}$$

15.

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x - \sin x \,.\cos x}{\cos x}}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x \;(1 - \cos x)}{\cos x}}{x^3}$$

$$= \lim_{x \to 0} \frac{\tan x \,(1 - \cos x)}{x^3}$$

$$= \lim_{x \to 0} \frac{\tan x \,.\, 2\sin^2 (x/2)}{x^3}$$

$$= \lim_{x \to 0} \frac{\tan x}{x} \,.\, \frac{2\sin^2 (x/2)}{x^2}$$ DISTRIBUTE

SQUARE–SQUARE THE WHOLE SQUARE

$$= \lim_{x \to 0} \frac{\tan x}{x} \,.\, 2\left(\frac{\sin (x/2)}{x}\right)^2$$

$$= \lim_{x \to 0} \frac{\tan x}{x} \,.2\left(\frac{1}{2}\frac{\sin (x/2)}{x/2}\right)^2$$

LIMIT JATHI VALUE AATI , KAHI FORMULA LAGTI KAIHI PASTE HOTI

$$= 1 \;.\; 2\left(\frac{1}{2}\,(1)\right)^2$$

$$= 1/2$$

SOLUTION TO Q SET - 2

01.

$$\lim_{x \to \pi/2} \frac{\cot^2 x}{1 - \sin x}$$

$$= \lim_{x \to \pi/2} \frac{\cos^2 x}{\sin^2 x \,.(1 - \sin x)}$$

$$= \lim_{x \to \pi/2} \frac{1 - \sin^2 x}{\sin^2 x \,.(1 - \sin x)}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{(1 - \sin x)}(1 + \sin x)}{\sin^2 x \,.\cancel{(1 - \sin x)}} \qquad \text{CUT}$$

$x \to \pi/2$; $\sin x \to \sin \pi/2$; $\sin x \to 1$,

$\sin x \neq 1$; $1 - \sin x \neq 0$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{\sin^2 x \,.} \qquad \text{COPY}$$

$$= \frac{1 + \sin \pi/2}{\sin^2 \pi/2} \qquad \text{PASTE}$$

$$= \frac{1 + 1}{1} = 2$$

02.

$$\lim_{x \to 0} \frac{\tan^2 x}{1 - \cos x}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\cos^2 x \,.(1 - \cos x)}$$

$$= \lim_{x \to 0} \frac{1 - \cos^2 x}{\cos^2 x \,.(1 - \cos x)}$$

$$= \lim_{x \to 0} \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cos^2 x \,.\cancel{(1 - \cos x)}} \qquad \text{CUT}$$

$x \to 0$; $\cos x \to \cos 0$; $\cos x \to 1$,

$\cos x \neq 1$; $1 - \cos x \neq 0$

$$= \lim_{x \to 0} \frac{1 + \cos x}{\cos^2 x \,.} \qquad \text{COPY}$$

$$= \frac{1 + \cos 0}{\cos^2 0} \qquad \text{PASTE}$$

$$= \frac{1 + 1}{1} = 2$$

03.

$$\lim_{x \to 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{\sqrt{3 + \cos x} - 2} \cdot \frac{\sqrt{3 + \cos x} + 2}{\sqrt{3 + \cos x} + 2}$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{3 + \cos x - 4} \cdot \frac{\sqrt{3 + \cos x} + 2}{1}$$

$$= \lim_{x \to 0} \frac{1 - \cos^2 x}{\cos x - 1} \cdot \frac{\sqrt{3 + \cos x} + 2}{1}$$

$$= \lim_{x \to 0} \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{-\cancel{(1 - \cos x)}} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \qquad \text{CUT}$$

$x \to 0$; $\cos x \to \cos 0$; $\cos x \to 1$,

$\cos x \neq 1$; $1 - \cos x \neq 0$

$$= \lim_{x \to 0} \frac{1 + \cos x}{-1} \cdot \frac{\sqrt{3 + \cos x} + 2}{1} \qquad \text{COPY}$$

$$= \lim_{x \to 0} \frac{1 + \cos 0}{-1} \cdot \frac{\sqrt{3 + \cos 0} + 2}{1} \qquad \text{PASTE}$$

$$= \frac{1 + 1}{-1} \,.\, \frac{\sqrt{3 + 1} + 2}{1}$$

$$= \frac{2}{-1} \cdot \frac{2+2}{1}$$

$$= -8$$

04.

$$\lim_{x \to \pi/2} \frac{\cos^2 x}{\sqrt{3+\sin x} - 2}$$

$$= \lim_{x \to \pi/2} \frac{\cos^2 x}{\sqrt{3+\sin x} - 2} \; \frac{\sqrt{3+\sin x} + 2}{\sqrt{3+\sin x} + 2}$$

$$= \lim_{x \to \pi/2} \frac{\cos^2 x}{3+\sin x - 4} \; \frac{\sqrt{3+\sin x} + 2}{1}$$

$$= \lim_{x \to \pi/2} \frac{1 - \sin^2 x}{\sin x - 1} \; \frac{\sqrt{3+\sin x} + 2}{1}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{(1-\sin x)}(1+\sin x)}{-\cancel{(1-\sin x)}} \; \frac{\sqrt{3+\sin x} + 2}{1}$$

CUT

$x \to \pi/2$; $\sin x \to \sin \pi/2$; $\sin x \to 1$,

$\sin x \neq 1$; $1 - \sin x \neq 0$

$$= \lim_{x \to \pi/2} \frac{1+\sin x}{-1} \quad \frac{\sqrt{3+\sin x} + 2}{1}$$

COPY

$$= \lim_{x \to \pi/2} \frac{1+\sin \pi/2}{-1} \quad \frac{\sqrt{3+\sin \pi/2} + 2}{1}$$

PASTE

$$= \frac{1+1}{-1} \cdot \frac{\sqrt{3+1} + 2}{1}$$

$$= \frac{2}{-1} \cdot \frac{2+2}{1}$$

$$= -8$$

05.

$$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{1 - \sin^2 x} \; \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \pi/2} \frac{2 - (1+\sin x)}{(1-\sin x)(1+\sin x)} \; \frac{1}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \pi/2} \frac{1 - \cancel{\sin x}}{(1-\cancel{\sin x})(1+\sin x)} \; \frac{1}{\sqrt{2} + \sqrt{1+\sin x}}$$

CUT

$x \to \pi/2$; $\sin x \to \sin \pi/2$; $\sin x \to 1$,

$\sin x \neq 1$; $1 - \sin x \neq 0$

COPY

$$= \lim_{x \to \pi/2} \frac{1}{1+\sin x} \; \frac{1}{\sqrt{2} + \sqrt{1+\sin x}}$$

PASTE

$$= \frac{1}{1+\sin \pi/2} \quad \frac{1}{\sqrt{2} + \sqrt{1+\sin \pi/2}}$$

$$= \frac{1}{1+1} \quad \frac{1}{\sqrt{2} + \sqrt{1+1}}$$

$$= \frac{1}{2} \quad \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

06.

$$\lim_{x \to \pi} \frac{\sqrt{1-\cos x} - \sqrt{2}}{\sin^2 x}$$

$$= \lim_{x \to \pi} \frac{\sqrt{1-\cos x} - \sqrt{2}}{1 - \cos^2 x} \; \frac{\sqrt{1-\cos x} + \sqrt{2}}{\sqrt{1-\cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{1 - \cos x - 2}{1 - \cos^2 x} \cdot \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{-\cos x - 1}{(1+\cos x)(1-\cos x)} \cdot \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \lim_{x \to \pi} \frac{-\cancel{(1 + \cos x)}}{\cancel{(1+\cos x)}(1-\cos x)} \cdot \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$x \to \pi$; $\cos x \to \cos \pi$; $\cos x \to -1$

$\cos x \neq -1 \;\therefore\; 1 + \cos x \neq 0$

$$= \lim_{x \to \pi} \frac{1}{1 - \cos x} \cdot \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}$$

$$= \frac{1}{1 - \cos \pi} \cdot \frac{1}{\sqrt{1 - \cos \pi} + \sqrt{2}}$$

$$= \frac{1}{1 - (-1)} \cdot \frac{1}{\sqrt{1 - (-1)} + \sqrt{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{4\sqrt{2}}$$

07.

$$\lim_{x \to 0} \frac{\sin^2 x}{1 - \cos^3 x}$$

$$= \lim_{x \to 0} \frac{1 - \cos^2 x}{(1 - \cos x)(1 + \cos x + \cos^2 x)}$$

$$= \lim_{x \to 0} \frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}(1 + \cos x + \cos^2 x)} \quad \text{CUY}$$

$x \to 0$; $\cos x \to \cos 0$; $\cos x \to 1$,

$\cos x \neq 1$; $1 - \cos x \neq 0$

$$= \lim_{x \to 0} \frac{1 + \cos x}{1 + \cos x + \cos^2 x} \quad \text{COPY}$$

$$= \frac{1 + \cos 0}{1 + \cos 0 + \cos^2 0} \quad \text{PASTE}$$

$$= \frac{1 + 1}{1 + 1 + 1} = \frac{2}{3}$$

08.

$$\lim_{x \to \pi} \frac{\sin^2 x}{1 + \cos^3 x}$$

$$= \lim_{x \to \pi} \frac{1 - \cos^2 x}{(1 + \cos x)(1 - \cos x + \cos^2 x)}$$

$$= \lim_{x \to \pi} \frac{\cancel{(1 + \cos x)}(1 - \cos x)}{\cancel{(1 + \cos x)}(1 - \cos x + \cos^2 x)} \quad \text{CUT}$$

$x \to \pi$; $\cos x \to \cos \pi$; $\cos x \to -1$,

$\cos x \neq -1$; $1 + \cos x \neq 0$

$$= \lim_{x \to \pi} \frac{1 - \cos x}{1 - \cos x + \cos^2 x} \quad \text{COPY}$$

$$= \frac{1 - \cos \pi}{1 - \cos \pi + \cos^2 \pi} \quad \text{PASTE}$$

$$= \frac{1 + 1}{1 + 1 + 1}$$

$$= 2/3$$

09.

$$\lim_{x \to \pi/2} \frac{1 - \sin^3 x}{\cos^2 x}$$

$$= \lim_{x \to \pi/2} \frac{(1 - \sin x)(1 + \sin x + \sin^2 x)}{1 - \sin^2 x}$$

$$= \lim_{x \to \pi/2} \frac{\cancel{(1 - \sin x)}(1 + \sin x + \sin^2 x)}{\cancel{(1 - \sin x)}(1 + \sin x)}$$

$x \to {}^{\pi}/_{2}$; $\sin x \to \sin {}^{\pi}/_{2}$; $\sin x \to 1$,

$\sin x \neq 1$; $1 - \sin x \neq 0$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x + \sin^2 x}{1 + \sin x}$$

$$= \frac{1 + \sin \frac{\pi}{2} + \sin^2 \frac{\pi}{2}}{1 + \sin \frac{\pi}{2}}$$

$$= \frac{1 + 1 + 1}{1 + 1} = \frac{3}{2}$$

10.

$$\lim_{x \to \pi/4} \frac{1 - \tan^3 x}{\sec^2 x - 2} \qquad \boxed{1 + \tan^2 x = \sec^2 x}$$

$$= \lim_{x \to \pi/4} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{1 + \tan^2 x - 2}$$

$$= \lim_{x \to \pi/4} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{\tan^2 x - 1}$$

$$= \lim_{x \to \pi/4} \frac{(1 - \tan x)(1 + \tan x + \tan^2 x)}{(\tan x - 1)(\tan x + 1)}$$

$$= \lim_{x \to \pi/4} \frac{\cancel{(1 - \tan x)}(1 + \tan x + \tan^2 x)}{-\cancel{(1 - \tan x)}(\tan x + 1)}$$

CUT

$x \to {}^{\pi}/_{4}$; $\tan x \to \tan {}^{\pi}/_{4}$; $\tan x \to 1$,

$\tan x \neq 1$; $1 - \tan x \neq 0$

$$= \lim_{x \to \pi/4} \frac{1 + \tan x + \tan^2 x}{-(\tan x + 1)}$$

COPY

$$= \frac{1 + \tan \frac{\pi}{4} + \tan^2 \frac{\pi}{4}}{-(\tan \frac{\pi}{4} + 1)}$$

PASTE

$$= \frac{1 + 1 + 1}{-(1 + 1)}$$

$$= -3/2$$

11.

$$\lim_{x \to \pi/4} \frac{\operatorname{cosec}^2 x - 2}{1 - \cot^3 x} \qquad \boxed{1 + \cot^2 x = \operatorname{cosec}^2 x}$$

$$= \lim_{x \to \pi/4} \frac{1 + \cot^2 x - 2}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \to \pi/4} \frac{\cot^2 x - 1}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \to \pi/4} \frac{(\cot x - 1)(\cot x + 1)}{(1 - \cot x)(1 + \cot x + \cot^2 x)}$$

$$= \lim_{x \to \pi/4} \frac{-\cancel{(1 - \cot x)}(\cot x + 1)}{\cancel{(1 - \cot x)}(1 + \cot x + \cot^2 x)}$$

$x \to {}^{\pi}/_{4}$; $\cot x \to \cot {}^{\pi}/_{4}$; $\cot x \to 1$,

$\cot x \neq 1$; $1 - \cot x \neq 0$

$$= \lim_{x \to \pi/4} \frac{-(\cot x + 1)}{1 + \cot x + \cot^2 x}$$

COPY

$$= \frac{-(\cot \frac{\pi}{4} + 1)}{1 + \cot \frac{\pi}{4} + \cot^2 \frac{\pi}{4}}$$

PASTE

$$= \frac{-(1 + 1)}{1 + 1 + 1}$$

$$= -2/3$$

12.

$$\lim_{x \to \pi/4} \frac{\cot x - 1}{2 - \operatorname{cosec}^2 x}$$

$$= \lim_{x \to \pi/4} \frac{\cot x - 1}{2 - (1 + \cot^2 x)}$$

$$= \lim_{x \to \pi/4} \frac{\cot x - 1}{2 - 1 - \cot^2 x}$$

$$= \lim_{x \to \pi/4} \frac{\cot x - 1}{1 - \cot^2 x}$$

$$= \lim_{x \to \pi/4} \frac{\cot x - 1}{(1 - \cot x)(1 + \cot x)}$$

$$= \lim_{x \to \pi/4} \frac{-(1 - \cot x)}{(1 - \cot x)(1 + \cot x)}$$ CUT

$x \to \pi/4$; $\cot x \to \cot \pi/4$; $\cot x \to 1$,

$\cot x \neq 1$; $1 - \cot x \neq 0$

$$= \lim_{x \to \pi/4} \frac{-1}{1 + \cot x}$$ COPY

$$= \frac{-1}{1 + \cot \pi/4}$$ PASTE

$$= \frac{-1}{1 + 1} = \frac{-1}{2}$$

13.

$$\lim_{x \to \pi/6} \frac{2\sin x - 1}{4\cos^2 x - 3}$$

$$= \lim_{x \to \pi/6} \frac{2\sin x - 1}{4(1 - \sin^2 x) - 3}$$

$$= \lim_{x \to \pi/6} \frac{2\sin x - 1}{4 - 4\sin^2 x - 3}$$

$$= \lim_{x \to \pi/6} \frac{2\sin x - 1}{1 - 4\sin^2 x}$$

$$= \lim_{x \to \pi/6} \frac{2\sin x - 1}{(1 - 2\sin x)(1 + 2\sin x)}$$

$$= \lim_{x \to \pi/6} \frac{-\cancel{(1 - 2\sin x)}}{\cancel{(1 - 2\sin x)}(1 + 2\sin x)}$$ CUT

$x \to \pi/6$; $\sin x \to \sin \pi/6$; $\sin x \to 1/2$,

$2\sin x \to 1$; $2\sin x \neq 1$; $1 - 2\sin x \neq 0$

$$= \lim_{x \to \pi/6} \frac{-1}{1 + 2\sin x}$$ COPY

$$= \lim_{x \to \pi/6} \frac{-1}{1 + 2\sin \pi/6}$$ PASTE

$$= \frac{-1}{1 + 2(1/2)}$$

$$= -1/2$$

14.

$$\lim_{x \to \pi/6} \frac{2 - \operatorname{cosec} x}{\cot^2 x - 3}$$

$$= \lim_{x \to \pi/6} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1 - 3}$$

$$= \lim_{x \to \pi/6} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 4}$$

$$= \lim_{x \to \pi/6} \frac{2 - \operatorname{cosec} x}{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}$$

$$= \lim_{x \to \pi/6} \frac{-\cancel{(\operatorname{cosec} x - 2)}}{\cancel{(\operatorname{cosec} x - 2)}(\operatorname{cosec} x + 2)}$$ CUT

$x \to \pi/6$; $\operatorname{cosec} x \to \operatorname{cosec} \pi/6$;

$\operatorname{cosec} x \to 2$, $\operatorname{cosec} x \neq 2$; $\operatorname{cosec} x - 2 \neq 0$

$$= \lim_{x \to \pi/6} \frac{-1}{\operatorname{cosec} x + 2}$$ COPY

$$= \frac{-1}{\operatorname{cosec} \pi/6 + 2}$$ PASTE

$$= \frac{-1}{2 + 2} = \frac{-1}{4}$$

15.

$$\lim_{x \to \pi/3} \frac{\sec^3 x - 8}{\tan^2 x - 3}$$

$$= \lim_{x \to \pi/3} \frac{\sec^3 x - 2^3}{\tan^2 x - 3}$$

$$= \lim_{x \to \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{\sec^2 x - 1 - 3}$$

$$= \lim_{x \to \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{\sec^2 x - 4}$$

$$= \lim_{x \to \pi/3} \frac{(\sec x - 2)(\sec^2 x + 2\sec x + 4)}{(\sec x - 2)(\sec x + 2)}$$

$x \to \pi/3$; $\sec x \to \sec \pi/3$; $\sec x \to 2$,

$\sec x \neq 2$; $\sec x - 2 \neq 0$

$$= \frac{\sec^2 \pi/3 + 2\sec \pi/3 + 4}{\sec \pi/3 + 2}$$

$$= \frac{2^2 + 2(2) + 4}{2 + 2}$$

$$= 3$$

SOLUTION TO Q SET - 3

01.

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \cdot \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \to \pi} \frac{2 + \cos x - 1}{(\pi - x)^2} \cdot \frac{1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2} \cdot \frac{1}{\sqrt{2 + \cos x} + 1}$$

Put $x = \pi + h$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi + h)}{\left[(\pi - (\pi + h)\right]^2} \cdot \frac{1}{\sqrt{2 + \cos(\pi + h)} + 1}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{(\pi - \pi - h)^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{h^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 (h/2)}{h^2} \cdot \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \to 0} 2\left(\frac{1}{2} \frac{\sin (h/2)}{h/2}\right)^2 \frac{1}{\sqrt{2 - \cos h} + 1}$$

$$= \lim_{h \to 0} 2\left(\frac{1}{2} (1)\right)^2 \frac{1}{\sqrt{2 - \cos 0} + 1}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{\sqrt{2 - 1} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{1 + 1} = 1/4$$

02.

$$\lim_{x \to \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2}$$

$$= \lim_{x \to \pi} \frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \cdot \frac{\sqrt{5 + \cos x} + 2}{\sqrt{5 + \cos x} + 2}$$

$$= \lim_{x \to \pi} \frac{5 + \cos x - 4}{(\pi - x)^2} \cdot \frac{1}{\sqrt{5 + \cos x} + 2}$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2} \cdot \frac{1}{\sqrt{5 + \cos x} + 2}$$

Put $x = \pi + h$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi + h)}{\left[(\pi - (\pi + h)\right]^2} \cdot \frac{1}{\sqrt{5 + \cos(\pi + h)} + 2}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{(\pi - \pi - h)^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{h^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 (h/2)}{h^2} \cdot \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \to 0} 2\left(\frac{1}{2} \frac{\sin (h/2)}{h/2}\right)^2 \frac{1}{\sqrt{5 - \cos h} + 2}$$

$$= \lim_{h \to 0} 2\left(\frac{1}{2} (1)\right)^2 \frac{1}{\sqrt{5 - \cos 0} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{\sqrt{5 - 1} + 2}$$

$$= 2 \times \frac{1}{4} \cdot \frac{1}{2 + 2} = 1/8$$

03.

$$\lim_{x \to \pi/4} \frac{1 - \tan x}{\pi - 4x}$$

$\boxed{\text{PUT } \quad x = \pi/4 + h}$

$$= \lim_{h \to 0} \frac{1 - \tan(\pi/4 + h)}{\pi - 4(\pi/4 + h)}$$

$$= \lim_{h \to 0} \frac{1 - \dfrac{\tan \pi/4 + \tan h}{1 - \tan \pi/4 \,.\, \tan h}}{\pi - \pi - 4h}$$

$$= \lim_{h \to 0} \frac{1 - \dfrac{1 + \tan h}{1 - \tan h}}{-4h}$$

$$= \lim_{h \to 0} \frac{1 - \tan h - 1 - \tan h}{-4h \,.\, 1 - \tan h}$$

$$= \lim_{h \to 0} \frac{-2\tan h}{-4h \,.\, 1 - \tan h}$$

$$= \lim_{h \to 0} \frac{2}{4} \frac{\tan h}{h} \frac{1}{1 - \tan h}$$

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$$= \frac{2}{4} \,.\, 1 \; \frac{1}{1 - \tan 0}$$

$$= \frac{1}{2}$$

04.

$$\lim_{x \to \pi/3} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

$\boxed{\text{PUT } \quad x = \pi/3 + h}$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \,.\, \tan h}}{\pi - \pi - 3h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - \dfrac{\sqrt{3} + \tan h}{1 - \sqrt{3}.\tan h}}{-3h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3}(1 - \sqrt{3}.\tan h) - \sqrt{3} - \tan h}{-3h \,.\, 1 - \sqrt{3}.\tan h}$$

$$= \lim_{h \to 0} \frac{\sqrt{3} - 3.\tan h - \sqrt{3} - \tan h}{-3h \,.\, 1 - \sqrt{3}.\tan h}$$

$$= \lim_{h \to 0} \frac{-4\tan h}{-3h \,.\, 1 - \sqrt{3}\tan h}$$

$$= \lim_{h \to 0} \frac{4}{3} \frac{\tan h}{h} \frac{1}{1 - \sqrt{3}\tan h}$$

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$$= \frac{4}{3} \,.\, 1 \; \frac{1}{1 - \sqrt{3} \,.\, \tan 0}$$

$$= \frac{4}{3}$$

05.

$$\lim_{x \to \pi/4} \frac{\cos x - \sin x}{\pi - 4x}$$

$$\boxed{\text{PUT} \quad x = \pi/4 + h}$$

$$= \lim_{h \to 0} \frac{\cos(\pi/4 + h) - \sin(\pi/4 + h)}{\pi - 4(\pi/4 + h)}$$

$$= \lim_{h\to 0} \frac{(\cos \pi/4 \cos h - \sin \pi/4 \,.\, \sin h) - (\sin \pi/4 \cosh + \cos \pi/4 \,.\, \sin h)}{\pi - \pi - 4h}$$

$$= \lim_{h\to 0} \frac{(1/\sqrt{2} \,.\, \cos h - 1/\sqrt{2} \,.\, \sin h) - (1/\sqrt{2} \,.\, \cos h + 1/\sqrt{2} \,.\, \sin h)}{-4h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{2}}\cosh - \frac{1}{\sqrt{2}}\sinh - \frac{1}{\sqrt{2}}\cosh - \frac{1}{\sqrt{2}}\sinh}{-4h}$$

$$= \lim_{h\to 0} \frac{-\frac{2}{\sqrt{2}}\sin h}{-4h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2} \,.\, \sin h}{4h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2}}{4} \,.\, \frac{\sin h}{h}$$

$$= \frac{\sqrt{2}}{4}$$

06.

$$\lim_{x \to 1} \frac{1 + \cos \pi x}{(1 - x)^2}$$

$$\boxed{\text{PUT} \quad x = 1 + h}$$

$$= \lim_{h \to 0} \frac{1 + \cos \pi(1 + h)}{[1 - (1 + h)]^2}$$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi + \pi h)}{(1 - 1 - h)^2}$$

$$= \lim_{h \to 0} \frac{1 - \cos \pi h}{h^2}$$

$$= \lim_{h\to 0} \frac{2\sin^2 \frac{\pi h}{2}}{h^2}$$

$$= \lim_{h\to 0} 2\left(\frac{\sin \frac{\pi h}{2}}{h}\right)^2$$

$$= \lim_{h\to 0} 2\left(\frac{\frac{\pi}{2} \sin \frac{\pi h}{2}}{\frac{\pi h}{2}}\right)^2$$

$$= 2\left(\frac{\pi}{2}(1)\right)^2$$

$$= 2\,\frac{\pi^2}{4}$$

$$= \frac{\pi^2}{2}$$

07.

$$\lim_{x \to \pi/2} \frac{\operatorname{cosec} x - 1}{(\pi/2 - x)^2}$$

$$= \lim_{x \to \pi/2} \frac{\frac{1}{\sin x} - 1}{(\pi/2 - x)^2}$$

$$= \lim_{x \to \pi/2} \frac{1 - \sin x}{(\pi/2 - x)^2 \sin x}$$

$$\boxed{\text{PUT } x = \pi/2 + h}$$

$$= \lim_{h \to 0} \frac{1 - \sin(\pi/2 + h)}{\left[\pi/2 - (\pi/2 + h)\right]^2 . \sin(\pi/2 + h)}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{[\pi/2 - \pi/2 - h)^2 . \cos h}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{h^2 . \cos h}$$

$$= \lim_{h \to 0} \frac{2 \sin^2 (h/2)}{h^2} \quad \frac{1}{\cos h}$$

$$= \lim_{h \to 0} 2 \frac{\sin^2 (h/2)}{h^2} \quad \frac{1}{\cos h}$$

$$= \lim_{h \to 0} 2 \left(\frac{\sin (h/2)}{h}\right)^2 \frac{1}{\cos h}$$

$$= \lim_{h \to 0} 2 \left(\frac{1}{2} \frac{\sin (h/2)}{h/2}\right)^2 \frac{1}{\cos h}$$

$$= 2 \left(\frac{1}{2} (1)\right)^2 \frac{1}{\cos 0}$$

$$= \frac{1}{2}$$

08.

$$\lim_{x \to \pi/2} \frac{3 \cos x + \cos 3x}{(\pi - 2x)^3}$$

$$= \lim_{x \to \pi/2} \frac{3 \cos x + (4 \cos^3 x - 3 \cos x)}{(\pi - 2x)^3}$$

$$= \lim_{x \to \pi/2} \frac{3 \cos x + 4 \cos^3 x - 3 \cos x}{(\pi - 2x)^3}$$

$$= \lim_{x \to \pi/2} \frac{4 \cos^3 x}{(\pi - 2x)^3}$$

$$\boxed{\text{PUT } x = \pi/2 + h}$$

$$= \lim_{h \to 0} \frac{4 \cos^3 (\pi/2 + h)}{\left[\pi - 2 (\pi/2 + h)\right]^3}$$

$\cos (90 + \theta) = - \sin \theta$

$$= \lim_{h \to 0} \frac{- 4 \sin^3 h}{(\pi - \pi - 2h)^2}$$

$$= \lim_{h \to 0} \frac{- 4 \sin^3 h}{- 8h^3}$$

$$= \lim_{h \to 0} \frac{4}{8} \frac{\sin^3 h}{h^3}$$

$$= \lim_{h \to 0} \frac{4}{8} \left(\frac{\sin h}{h}\right)^3$$

$$= \frac{4}{8} (1)^3$$

$$= \frac{1}{2}$$

Q SET - 1

BASED ON $\lim_{X \to 0} \left[1 + x\right]^{1/x} = e$

01. $\lim_{x \to 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}}$ ans : $e^{1/3}$

02. $\lim_{x \to 0} \left(1 + \frac{x}{4}\right)^{\frac{1}{x}}$ ans : $e^{1/4}$

03. $\lim_{x \to 0} \left(1 + \frac{2x}{3}\right)^{\frac{1}{x}}$ ans : $e^{2/3}$

04. $\lim_{x \to 0} \left(1 + \frac{3x}{4}\right)^{\frac{1}{x}}$ ans : $e^{3/4}$

05. $\lim_{x \to 0} (1 + 2x)^{\frac{3}{x}}$ ans : e^6

06. $\lim_{x \to 0} (1 + 2x)^{\frac{5}{x}}$ ans : e^{10}

07. $\lim_{x \to 0} \left[\frac{1 + 4x}{1 - 4x}\right]^{\frac{1}{x}}$ ans : e^8

08. $\lim_{x \to 0} \left[\frac{1 - 5x}{1 + 5x}\right]^{\frac{1}{x}}$ ans : e^{-10}

09. $\lim_{x \to 0} \left[\frac{1 + 8x}{1 - 8x}\right]^{\frac{1}{x}}$ ans : e^{16}

10. $\lim_{x \to 0} \left[\frac{1 - 3x}{1 + 4x}\right]^{\frac{1}{x}}$ ans : e^{-7}

11. $\lim_{x \to 0} \left[\frac{4 + x}{4 - x}\right]^{\frac{1}{x}}$ ans : $e^{1/2}$

Q SET - 2

BASED ON $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

01. $\lim_{x\to0} \frac{\log(1 + 2x)}{x}$ ans : 2

02. $\lim_{x\to0} \frac{\log(1 + 6x)}{2x}$ ans : 3

03. $\lim_{x \to 0} \frac{1}{x} \log\left[1 + \frac{8x}{3}\right]$ ans : $8/3$

04. $\lim_{x \to 0} \frac{\log(1 + 5x) - \log(1 + 3x)}{x}$ ans : 2

05. $\lim_{x \to 0} \frac{\log 7 + \log\left(\frac{x + 1}{7}\right)}{x}$ ans : 1

06. $\lim_{x \to 0} \frac{\log(2 + x) - \log 2}{x}$ ans : $1/2$

Q SET - 3

BASED ON $\lim_{x \to 0} \frac{a^x - 1}{x} = \log a$

01. $\lim_{x \to 0} \frac{5^x - 4^x}{x}$ ans: $\log\left[\frac{5}{4}\right]$

02. $\lim_{x \to 0} \frac{4^x - 3^x}{5^x - 1}$ ans: $\frac{\log\left[\frac{4}{3}\right]}{\log 5}$

03. $\lim_{x \to 0} \frac{6^x - 3^x}{4^x - 1}$ ans : $1/2$

04. $\lim_{x \to 0} \dfrac{5^x - 3^x}{4^x - 2^x}$ ans: $\dfrac{\log\left(\frac{5}{3}\right)}{\log 2}$

05. $\lim_{x \to 0} \dfrac{a^{2x} - b^x}{x}$ ans: $\log\left(\frac{a^2}{b}\right)$

06. $\lim_{x \to 0} \dfrac{2^x + 3^x + 4^x - 3}{x}$

ans : log 24

07. $\lim_{x \to 0} \dfrac{2^x + 5^x + 7^x - 3}{x}$

ans : log 70

08. $\lim_{x \to 0} \dfrac{5^x + 3^x - 2^x - 1}{x}$

ans: $\log\left(\frac{15}{2}\right)$

09. $\lim_{x \to 0} \dfrac{4^x + 5^x - 2^{x+1}}{x}$

ans : log 5

10. $\lim_{x \to 0} \dfrac{a^x + b^x - 2^{x+1}}{x}$

ans: $\log\left(\frac{ab}{4}\right)$

11. $\lim_{x \to 0} \dfrac{2^x + 3^x + 4^x - 3^{x+1}}{x}$

ans: $\log\left(\frac{8}{9}\right)$

Q SET - 4

01. $\lim_{x \to 0} \dfrac{10^x - 5^x - 2^x + 1}{x^2}$

ans : log2 . log5

02. $\lim_{x \to 0} \dfrac{15^x - 3^x - 5^x + 1}{x^2}$

ans :log 3 . log 5

03. $\lim_{x \to 0} \dfrac{21^x - 3^x - 7^x + 1}{x^2}$

ans :log 3 . log 7

04. $\lim_{x \to 0} \dfrac{15^x - 3^x - 5^x + 1}{x.\tan x}$

ans :log 3 . log 5

05. $\lim_{x \to 0} \dfrac{12^x - 3^x - 4^x + 1}{1 - \cos 2x}$

ans :log 3 . log 2

Q SET - 5

01. $\lim_{x \to 0} \dfrac{a^x + a^{-x} - 2}{x^2}$ ans : $(\log a)^2$

02. $\lim_{x \to 0} \dfrac{5^x + 5^{-x} - 2}{x^2}$ ans : $(\log 5)^2$

03. $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{\cos 3x - \cos 5x}$ ans : $^1/_8$

04. $\lim_{x \to 0} \dfrac{5^x + 5^{-x} - 2}{\cos 2x - \cos 6x}$ ans : $\dfrac{(\log 5)^2}{16}$

Q SET - 6

01. $\lim_{x \to 3} (x-2)^{\frac{1}{(x-3)}}$ ans : e

02. $\lim_{x \to 4} (x-3)^{\frac{1}{(x-4)}}$ ans : e

03. $\lim_{x \to 3} \dfrac{\log x - \log 3}{x - 3}$ ans : $^1/_3$

04. $\lim_{x \to 3} \dfrac{\log x - \log 3}{x^2 - 9}$ ans : $^1/_{18}$

SOLUTION TO Q SET - 1

01. $$\lim_{x \to 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{3}}$$

02. $$\lim_{x \to 0} \left(1 + \frac{x}{4}\right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left\{\left(1 + \frac{x}{4}\right)^{\frac{1}{\frac{x}{4}}}\right\}^{\frac{1}{4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{4}}$$

03. $$\lim_{x \to 0} \left(1 + \frac{2x}{3}\right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left\{\left(1 + \frac{2x}{3}\right)^{\frac{1}{\frac{2x}{3}}}\right\}^{\frac{2}{3}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{2}{3}}$$

04. $$\lim_{x \to 0} \left(1 + \frac{3x}{4}\right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left\{\left(1 + \frac{3x}{4}\right)^{\frac{1}{\frac{3x}{4}}}\right\}^{\frac{3}{4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{3}{4}}$$

05. $$\lim_{x \to 0} (1 + 2x)^{\frac{3}{x}}$$

$$= \lim_{x \to 0} \left((1 + 2x)^{\frac{1}{2x}}\right)^{2.3}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{6}$$

06. $$\lim_{x \to 0} (1 + 2x)^{\frac{5}{x}}$$

$$= \lim_{x \to 0} \left((1 + 2x)^{\frac{1}{2x}}\right)^{2.5}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{10}$$

07.

$$\lim_{x \to 0} \left(\frac{1 + 4x}{1 - 4x}\right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \to 0} \frac{(1 + 4x)^{\frac{1}{x}}}{(1 - 4x)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \to 0} \frac{\left((1 + 4x)^{\frac{1}{4x}}\right)^{4}}{\left((1 - 4x)^{\frac{-1}{4x}}\right)^{-4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{4}}{e^{-4}}$$

$$= e^{4 + 4} = e^{8}$$

08.

$$\lim_{x \to 0} \left(\frac{1 - 5x}{1 + 5x}\right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \to 0} \frac{(1 - 5x)^{\frac{1}{x}}}{(1 + 5x)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \to 0} \frac{\left((1 - 5x)^{\frac{-1}{5x}}\right)^{-5}}{\left((1 + 5x)^{\frac{1}{5x}}\right)^{5}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{-5}}{e^{5}}$$

$$= e^{-5 - 5} = e^{-10}$$

09.

$$\lim_{x \to 0} \left(\frac{1 + 8x}{1 - 8x}\right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \to 0} \frac{(1 + 8x)^{\frac{1}{x}}}{(1 - 8x)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \to 0} \frac{\left((1 + 8x)^{\frac{1}{8x}}\right)^{8}}{\left((1 - 8x)^{\frac{-1}{8x}}\right)^{-8}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{8}}{e^{-8}}$$

$$= e^{8 + 8} = e^{16}$$

10.

$$\lim_{x \to 0} \left(\frac{1 - 3x}{1 + 4x}\right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \to 0} \frac{(1 - 3x)^{\frac{1}{x}}}{(1 + 4x)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \to 0} \frac{\left((1 - 3x)^{\frac{-1}{3x}}\right)^{-3}}{\left((1 + 4x)^{\frac{1}{4x}}\right)^{4}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= \frac{e^{-3}}{e^{4}}$$

$$= e^{-3 - 4} = e^{-7}$$

11.
$$\lim_{x \to 0} \left(\frac{4 + x}{4 - x} \right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left(\frac{\frac{4 + x}{4}}{\frac{4 - x}{4}} \right)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left(\frac{1 + \frac{x}{4}}{1 - \frac{x}{4}} \right)^{\frac{1}{x}}$$

SPLIT THE POWER

$$= \lim_{x \to 0} \frac{\left(1 + \frac{x}{4}\right)^{\frac{1}{x}}}{\left(1 - \frac{x}{4}\right)^{\frac{1}{x}}}$$

SET THE POWER

$$= \lim_{x \to 0} \frac{\left\{ \left(1 + \frac{x}{4}\right)^{\frac{1}{\frac{x}{4}}} \right\}^{\frac{1}{4}}}{\left\{ \left(1 - \frac{x}{4}\right)^{\frac{1}{-\frac{x}{4}}} \right\}^{\frac{-1}{4}}}$$

$$= \frac{e^{\frac{1}{4}}}{e^{\frac{-1}{4}}}$$

LIMIT JATHI FORMULA AATI , INSIDE THING 'e' HO JATHI

$$= e^{\frac{1}{4} + \frac{1}{4}}$$

$$= e^{\frac{1}{2}}$$

SOLUTION TO Q SET - 2

01. $$\lim_{x\to0} \frac{\log(1 + 2x)}{x}$$

$$= \lim_{x \to 0} 2 \frac{\log(1 + 2x)}{2x}$$

LIMIT JATHI FORMULA AATI

$$= 2(1)$$

$$= 2$$

02. $$\lim_{x\to0} \frac{\log(1 + 6x)}{2x}$$

$$= \lim_{x \to 0} \frac{1}{2} \frac{\log(1 + 6x)}{x}$$

$$= \lim_{x \to 0} \frac{6}{2} \frac{\log(1 + 6x)}{6x}$$

LIMIT JATHI FORMULA AATI

$$= 3(1)$$

$$= 3$$

03. $$\lim_{x \to 0} \frac{1}{x} \log\left(1 + \frac{8x}{3}\right)$$

$$= \lim_{x \to 0} \frac{\log\left(1 + \frac{8x}{3}\right)}{x}$$

$$= \lim_{x \to 0} \frac{\frac{8}{3} \log\left(1 + \frac{8x}{3}\right)}{\frac{8x}{3}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{8}{3}(1)$$

$$= \frac{8}{3}$$

04. $\lim_{x \to 0} \dfrac{\log(1+5x) - \log(1+3x)}{x}$

$= \lim_{x \to 0} \dfrac{\log(1+5x)}{x} - \dfrac{\log(1+3x)}{x}$

$= \lim_{x \to 0} 5\,\dfrac{\log(1+5x)}{5x} - 3\,\dfrac{\log(1+3x)}{3x}$

LIMIT JATHI FORMULA AATI

$= 5(1) - 3(1) = 2$

05.

$= \lim_{x \to 0} \dfrac{\log 7 + \log\left(\dfrac{x+1}{7}\right)}{x}$

$= \lim_{x \to 0} \dfrac{\log\left(7 \cdot \dfrac{x+1}{7}\right)}{x}$

$= \lim_{x \to 0} \dfrac{\log(1+x)}{x}$

LIMIT JATHI FORMULA AATI

$= 1$

06. $\lim_{x \to 0} \dfrac{\log(2+x) - \log 2}{x}$

$= \lim_{x \to 0} \dfrac{\log\left[\dfrac{2+x}{2}\right]}{x}$

$= \lim_{x \to 0} \dfrac{\log\left[1 + \dfrac{x}{2}\right]}{x}$

$= \lim_{x \to 0} \dfrac{\dfrac{1}{2}\log\left[1 + \dfrac{x}{2}\right]}{\dfrac{x}{2}}$

LIMIT JATHI FORMULA AATI

$= \dfrac{1}{2}(1) = \dfrac{1}{2}$

SOLUTION TO Q SET - 3

01. $\lim_{x \to 0} \dfrac{5^x - 4^x}{x}$

$= \lim_{x \to 0} \dfrac{5^x - 1 - 4^x + 1}{x}$

$= \lim_{x \to 0} \dfrac{5^x - 1 - (4^x - 1)}{x}$

$= \lim_{x \to 0} \dfrac{5^x - 1}{x} - \dfrac{4^x - 1}{x}$

LIMIT JATHI FORMULA AATI

$= \log 5 - \log 4$

$= \log\left(\dfrac{5}{4}\right)$

02. $\lim_{x \to 0} \dfrac{4^x - 3^x}{5^x - 1}$

$= \lim_{x \to 0} \dfrac{4^x - 1 - 3^x + 1}{5^x - 1}$

$= \lim_{x \to 0} \dfrac{4^x - 1 - (3^x - 1)}{5^x - 1}$

Dividing Numerator & Denominator by x , as $x \to 0$, $x \neq 0$

$= \lim_{x \to 0} \dfrac{\dfrac{4^x - 1 - (3^x - 1)}{x}}{\dfrac{5^x - 1}{x}}$

$= \lim_{x \to 0} \dfrac{\dfrac{4^x - 1}{x} - \dfrac{3^x - 1}{x}}{\dfrac{5^x - 1}{x}}$

LIMIT JATHI FORMULA AATI

$= \dfrac{\log 4 - \log 3}{\log 5}$

$$= \frac{\log\left(\frac{4}{3}\right)}{\log 5}$$

03. $\lim_{x \to 0} \frac{6^x - 3^x}{4^x - 1}$

$$= \lim_{x \to 0} \frac{6^x - 1 - 3^x + 1}{4^x - 1}$$

$$= \lim_{x \to 0} \frac{6^x - 1 - (3^x - 1)}{4^x - 1}$$

Dividing Numerator & Denominator by x ,
as $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{6^x - 1 - (3^x - 1)}{x}}{\frac{4^x - 1}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{6^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{4^x - 1}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 6 - \log 3}{\log 4}$$

$$= \frac{\log\left(\frac{6}{3}\right)}{\log 4}$$

$$= \frac{\log 2}{\log 4}$$

$$= \frac{\log 2}{\log 2^2}$$

$$= \frac{\log 2}{2\log 2}$$

$$= 1/2$$

04. $\lim_{x \to 0} \frac{5^x - 3^x}{4^x - 2^x}$

$$= \lim_{x \to 0} \frac{5^x - 1 - 3^x + 1}{4^x - 1 - 2^x + 1}$$

$$= \lim_{x \to 0} \frac{5^x - 1 - (3^x - 1)}{4^x - 1 - (2^x - 1)}$$

Dividing Numerator & Denominator by x ,
as $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{5^x - 1 - (3^x - 1)}{x}}{\frac{4^x - 1 - (2^x - 1)}{x}}$$

$$= \lim_{x \to 0} \frac{\frac{5^x - 1}{x} - \frac{3^x - 1}{x}}{\frac{4^x - 1}{x} - \frac{2^x - 1}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 5 - \log 3}{\log 4 - \log 2}$$

$$= \frac{\log\left(\frac{5}{3}\right)}{\log\left(\frac{4}{2}\right)}$$

$$= \frac{\log\left(\frac{5}{3}\right)}{\log 2}$$

05. $\lim_{x \to 0} \frac{a^{2x} - b^x}{x}$

$$= \lim_{x \to 0} \frac{a^{2x} - 1 - b^x + 1}{x}$$

$$= \lim_{x \to 0} \frac{a^{2x} - 1 - (b^x - 1)}{x}$$

$$= \lim_{x \to 0} \frac{a^{2x}-1}{x} - \frac{b^x-1}{x}$$

$$= \lim_{x \to 0} 2\,\frac{a^{2x}-1}{2x} - \frac{b^x-1}{x}$$

LIMIT JATHI FORMULA AATI

$$= 2 \log a - \log b$$

$$= \log a^2 - \log b$$

$$= \log\left(\frac{a^2}{b}\right)$$

06. $$\lim_{x \to 0} \frac{2^x + 3^x + 4^x - 3}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \frac{4^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 3 + \log 4$$

$$= \log (2 \times 3 \times 4)$$

$$= \log 24$$

07. $$\lim_{x \to 0} \frac{2^x + 5^x + 7^x - 3}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1 + 5^x - 1 + 7^x - 1}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1}{x} + \frac{5^x - 1}{x} + \frac{7^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 5 + \log 7$$

$$= \log (2 \times 5 \times 7)$$

$$= \log 70$$

08. $$\lim_{x \to 0} \frac{5^x + 3^x - 2^x - 1}{x}$$

EK KA BHALA SUM SE KAR,DO KA BHALA KHUD SE KAR

$$= \lim_{x \to 0} \frac{5^x - 1 + 3^x - 2^x}{x}$$

$$= \lim_{x \to 0} \frac{5^x - 1 + 3^x - 1 - 2^x + 1}{x}$$

$$= \lim_{x \to 0} \frac{5^x - 1 + 3^x - 1 - (2^x - 1)}{x}$$

$$= \lim_{x \to 0} \frac{5^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 5 + \log 3 - \log 2$$

$$= \log\left(\frac{5 \times 3}{2}\right)$$

$$= \log\left(\frac{15}{2}\right)$$

09. $$\lim_{x \to 0} \frac{4^x + 5^x - 2^{x+1}}{x}$$

$$= \lim_{x \to 0} \frac{4^x + 5^x - 2^x\,2}{x}$$

$$= \lim_{x \to 0} \frac{4^x - 1 + 5^x - 1 - 2^x\,2 + 2}{x}$$

$$= \lim_{x \to 0} \frac{4^x - 1 + 5^x - 1 - 2(2^x - 1)}{x}$$

$$= \lim_{x \to 0} \frac{4^x - 1}{x} + \frac{5^x - 1}{x} - 2\frac{(2^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 4 + \log 5 - 2\log 2$$

$$= \log 4 + \log 5 - \log 4$$

$$= \log 5$$

10. $$\lim_{x \to 0} \frac{a^x + b^x - 2^{x+1}}{x}$$

$$= \lim_{x \to 0} \frac{a^x + b^x - 2^x\,2}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1 + b^x - 1 - 2^x\,2 + 2}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1 + b^x - 1 - 2(2^x - 1)}{x}$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} + \frac{b^x - 1}{x} - 2\frac{(2^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log a + \log b - 2\log 2$$

$$= \log a + \log b - \log 4$$

$$= \log\left(\frac{ab}{4}\right)$$

11. $$\lim_{x \to 0} \frac{2^x + 3^x + 4^x - 3^{x+1}}{x}$$

$$= \lim_{x \to 0} \frac{2^x + 3^x + 4^x - 3^x\,3}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1 - 3^x\,3 + 3}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1 + 3^x - 1 + 4^x - 1 - 3(3^x - 1)}{x}$$

$$= \lim_{x \to 0} \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \frac{4^x - 1}{x} - 3\frac{(3^x - 1)}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 2 + \log 3 + \log 4 - 3\log 3$$

$$= \log 2 + \log 3 + \log 4 - \log 27$$

$$= \log\left(\frac{2 \times 3 \times 4}{27}\right)$$

$$= \log\left(\frac{8}{9}\right)$$

SOLUTION TO Q SET - 4

01. $$\lim_{x \to 0} \frac{10^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x\,2^x - 5^x - 2^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x(2^x - 1) - 1(2^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)(2^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x - 1}{x} \cdot \frac{2^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 5 \,.\, \log 2$$

02. $$\lim_{x \to 0} \frac{15^x - 3^x - 5^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{5^x\,3^x - 3^x - 5^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x (5^x - 1) - 1(5^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(5^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} . \frac{5^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 3 . \log 5$$

03. $$\lim_{x \to 0} \frac{21^x - 3^x - 7^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{7^x 3^x - 3^x - 7^x + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x (7^x - 1) - 1(7^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(7^x - 1)}{x^2}$$

$$= \lim_{x \to 0} \frac{3^x - 1}{x} . \frac{7^x - 1}{x}$$

LIMIT JATHI FORMULA AATI

$$= \log 3 . \log 7$$

04. $$\lim_{x \to 0} \frac{15^x - 3^x - 5^x + 1}{x.\tan x}$$

$$= \lim_{x \to 0} \frac{5^x 3^x - 3^x - 5^x + 1}{x.\tan x}$$

$$= \lim_{x \to 0} \frac{3^x (5^x - 1) - 1(5^x - 1)}{x.\tan x}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(5^x - 1)}{x.\tan x}$$

Dividing Numerator & Denominator by x^2 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(3^x - 1)(5^x - 1)}{x^2}}{\frac{x.\tan x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\frac{3^x - 1}{x} . \frac{5^x - 1}{x}}{\frac{\tan x}{x}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{\log 3 . \log 5}{(1)}$$

$$= \log 3 . \log 5$$

05. $$\lim_{x \to 0} \frac{12^x - 3^x - 4^x + 1}{1 - \cos 2x}$$

$$= \lim_{x \to 0} \frac{4^x 3^x - 3^x - 4^x + 1}{2 \sin^2 x}$$

$$= \lim_{x \to 0} \frac{3^x (4^x - 1) - 1(4^x - 1)}{2 \sin^2 x}$$

$$= \lim_{x \to 0} \frac{(3^x - 1)(4^x - 1)}{2 \sin^2 x}$$

Dividing Numerator & Denominator by x^2 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(3^x - 1)(4^x - 1)}{x^2}}{\frac{2 \sin^2 x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\frac{3^x - 1}{x} . \frac{4^x - 1}{x}}{2 \left(\frac{\sin x}{x}\right)^2}$$

DISTRIBUTE

SQUARE SQUARE THE WHOLE SQUARE

LIMIT JATHI FORMULA AATI

$$= \frac{\log 3 \cdot \log 4}{2(1)^2}$$

$$= \frac{\log 3 \cdot 2\log 2}{2}$$

$$= \log 3 \cdot \log 2$$

SOLUTION TO Q SET - 5

01. $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$

$$= \lim_{x \to 0} \frac{a^x + \frac{1}{a^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(a^x)^2 + 1 - 2.a^x}{a^x \cdot x^2}$$

$$= \lim_{x \to 0} \frac{(a^x - 1)^2}{x^2} \frac{1}{a^x}$$

$$= \lim_{x \to 0} \left[\frac{a^x - 1}{x}\right]^2 \cdot \frac{1}{a^x}$$ SQUARE SQUARE THE WHOLE SQUARE

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= (\log a)^2 \cdot \frac{1}{a^0}$$

$$= (\log a)^2$$

02. $\lim_{x \to 0} \frac{5^x + 5^{-x} - 2}{x^2}$

$$= \lim_{x \to 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x \to 0} \frac{(5^x)^2 + 1 - 2.5^x}{5^x \cdot x^2}$$

$$= \lim_{x \to 0} \frac{(5^x - 1)^2}{x^2} \frac{1}{5^x}$$

$$= \lim_{x \to 0} \left[\frac{5^x - 1}{x}\right]^2 \cdot \frac{1}{5^x}$$ SQUARE SQUARE THE WHOLE SQUARE

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= (\log 5)^2 \cdot \frac{1}{5^0}$$

$$= (\log 5)^2$$

03. $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{\cos 3x - \cos 5x}$

$$= \lim_{x \to 0} \frac{e^x + \frac{1}{e^x} - 2}{-2\sin\left[\frac{3x+5x}{2}\right] \cdot \sin\left[\frac{3x-5x}{2}\right]}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x)^2 + 1 - 2.e^x}{e^x}}{-2\sin 4x \cdot \sin(-x)}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{e^x}}{2\sin 4x \cdot \sin x}$$

Dividing Numerator & Denominator by x^2 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{e^x \cdot x^2}}{\frac{2\sin 4x \cdot \sin x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\frac{(e^x - 1)^2}{x^2} \frac{1}{e^x}}{2\frac{\sin 4x}{x} \cdot \frac{\sin x}{x}}$$ DISTRIBUTE

$$= \lim_{x \to 0} \frac{\left[\frac{e^x - 1}{x}\right]^2 \frac{1}{e^x}}{2.4\frac{\sin 4x}{4x} \cdot \frac{\sin x}{x}}$$ SQUARE SQUARE THE WHOLE SQUARE

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= \frac{(\log e)^2 \cdot \frac{1}{e^0}}{2.4.(1).(1)} = 1/8$$

04. $\lim_{x \to 0} \dfrac{5^x + 5^{-x} - 2}{\cos 2x - \cos 6x}$

$$= \lim_{x \to 0} \frac{5^x + \frac{1}{5^x} - 2}{-2 \sin\left(\frac{2x+6x}{2}\right) . \sin\left(\frac{2x-6x}{2}\right)}$$

$$= \lim_{x \to 0} \frac{\frac{(5^x)^2 + 1 - 2.5^x}{5^x}}{- 2 \sin 4x \ . \ \sin(-2x)}$$

$$= \lim_{x \to 0} \frac{\frac{(5^x - 1)^2}{5^x}}{2 \sin 4x \ . \ \sin 2x}$$

Dividing Numerator & Denominator by x^2 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \frac{\frac{(5^x - 1)^2}{5^x . x^2}}{\frac{2 \sin 4x \ . \ \sin 2x}{x^2}}$$

$$= \lim_{x \to 0} \frac{\frac{(5^x - 1)^2}{x^2} \ \frac{1}{5^x}}{2 \frac{\sin 4x}{x} . \frac{\sin 2x}{x}}$$

DISTRIBUTE

$$= \lim_{x \to 0} \frac{\left(\frac{5^x - 1}{x}\right)^2 \frac{1}{5^x}}{2.4 \frac{\sin 4x}{4x} .2 \frac{\sin 2x}{2x}}$$

SQUARE SQUARE

THE WHOLE SQUARE

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= \frac{(\log 5)^2 . \frac{1}{5^0}}{2.4.(1).2(1)}$$

$$= \frac{(\log 5)^2}{16}$$

SOLUTION TO Q SET - 6

01. $\lim_{x \to 3} (x - 2)^{\frac{1}{(x - 3)}}$

$x = 3 + h$

$$= \lim_{h \to 0} (3 + h - 2)^{\frac{1}{3 + h - 3}}$$

$$= \lim_{h \to 0} (1 + h)^{\frac{1}{h}}$$

$= e$

02. $\lim_{x \to 4} (x - 3)^{\frac{1}{(x - 4)}}$

$x = 4 + h$

$$= \lim_{h \to 0} (4 + h - 3)^{\frac{1}{4 + h - 4}}$$

$$= \lim_{h \to 0} (1 + h)^{\frac{1}{h}} = e$$

03. $\lim_{x \to 3} \dfrac{\log x - \log 3}{x - 3}$

$x = 3 + h$

$$= \lim_{h \to 0} \frac{\log (3 + h) - \log 3}{3 + h - 3}$$

$$= \lim_{h \to 0} \frac{\log\left(\frac{3 + h}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(1 + \frac{h}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3} \log \left(1 + \frac{h}{3}\right)}{\frac{h}{3}}$$

LIMIT JATHI FORMULA AATI

$$= \frac{1}{3} (1)$$

$$= \frac{1}{3}$$

04. $$\lim_{x \to 3} \frac{\log x - \log 3}{x^2 - 9}$$

$$= \lim_{x \to 3} \frac{\log x - \log 3}{x - 3} \quad \frac{1}{x + 3}$$

$x = 3 + h$

$$= \lim_{h \to 0} \frac{\log (3 + h) - \log 3}{3 + h - 3} \quad \frac{1}{6 + h}$$

$$= \lim_{h \to 0} \frac{\log \left(\frac{3 + h}{3}\right)}{h} \quad \frac{1}{6 + h}$$

$$= \lim_{h \to 0} \frac{\log \left(1 + \frac{h}{3}\right)}{h} \quad \frac{1}{6 + h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3} \log \left(1 + \frac{h}{3}\right)}{\frac{h}{3}} \quad \frac{1}{6 + h}$$

LIMIT JATHI , KAHI FORMULA LAGTI , KAHI PASTE HOTI

$$= \frac{1}{3} (1) \quad \frac{1}{6 + 0}$$

$$= \frac{1}{18}$$